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SOLVED PAPER

Held on
24th Feb, Morning Shift

JEE MAIN 2021

SECTION-A (MULTIPLE CHOICE QUESTIONS)

1. A man is walking on a straight line. The arithmetic mean of the reciprocals of the intercepts of this line on the coordinate axes is $\frac{1}{4}$. Three stones A, B and C are placed at the points (1, 1), (2, 2) and (4, 4) respectively. Then which of these stones is/are on the path of the man?

(a) C only (b) A only
(c) B only (d) All the three

2. The area (in sq. units) of the part of the circle $x^2+y^2=36$, which is outside the parabola $y^2=9x$, is

(a) $12\pi-3\sqrt{3}$ (b) $24\pi+3\sqrt{3}$
(c) $12\pi+3\sqrt{3}$ (d) $24\pi-3\sqrt{3}$

3. If $e^{(\cos^2 x+\cos^4 x+\cos^6 x+\ldots\infty)\log_e 2}$ satisfies the equation $t^2-9t+8=0$, then the value of $\frac{2\sin x}{\sin x+\sqrt{3}\cos x}$ $\left(0<x<\frac{\pi}{2}\right)$ is

(a) $\frac{3}{2}$ (b) $\frac{1}{2}$ (c) $\sqrt{3}$ (d) $2\sqrt{3}$

4. A scientific committee is to be formed from 6 Indians and 8 foreigners, which includes at least 2 Indians and double the number of foreigners as Indians. Then the number of ways, the committee can be formed, is

(a) 560 (b) 1050 (c) 1625 (d) 575

5. The population $P=P(t)$ at time 't' of a certain species follows the differential equation $\frac{dP}{dt}=0.5P-450$. If $P(0)=850$, then the time at which population becomes zero is

(a) $2\log_e 18$ (b) $\log_e 9$
(c) $\frac{1}{2}\log_e 18$ (d) $\log_e 18$

6. If $\int\frac{\cos x-\sin x}{\sqrt{8-\sin 2x}}dx=a\sin^{-1}\left(\frac{\sin x+\cos x}{b}\right)+c$, where c is a constant of integration, then the ordered pair (a, b) is equal to

(a) (1, −3) (b) (3, 1) (c) (1, 3) (d) (−1, 3)

7. The function

$$f(x)=\frac{4x^3-3x^2}{6}-2\sin x+(2x-1)\cos x$$

(a) increases in $\left[\frac{1}{2},\infty\right)$ (b) decreases in $\left(-\infty,\frac{1}{2}\right]$
(c) decreases in $\left[\frac{1}{2},\infty\right)$ (d) increases in $\left(-\infty,\frac{1}{2}\right]$

8. The statement among the following that is a tautology is

(a) $B\to[A\wedge(A\to B)]$ (b) $[A\wedge(A\to B)]\to B$
(c) $A\vee(A\wedge B)$ (d) $A\wedge(A\vee B)$

9. $\lim\limits_{x\to 0}\frac{\int_0^{x^2}(\sin\sqrt{t})dt}{x^3}$ is equal to

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{1}{15}$ (d) 0

10. If $f: R\to R$ is a function defined by $f(x)=[x-1]\cos\left(\frac{2x-1}{2}\right)\pi$, where $[\cdot]$ denotes the greatest intger function, then f is

(a) continuous for every real x

JEE Main - 2021

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(b) discontinuous at all integral values of x except at $x = 1$

(c) continuous only at $x = 1$

(d) discontinuous only at $x = 1$

11. The distance of the point (1, 1, 9) from the point of intersection of the line $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ and the plane $x + y + z = 17$ is

(a) $\sqrt{38}$ (b) 38 (c) $2\sqrt{19}$ (d) $19\sqrt{2}$

12. The system of linear equations

$$3x - 2y - kz = 10$$
$$2x - 4y - 2z = 6$$
$$x + 2y - z = 5m$$

is inconsistent if

(a) $k = 3, m \neq \frac{4}{5}$ (b) $k \neq 3, m \neq \frac{4}{5}$

(c) $k = 3, m = \frac{4}{5}$ (d) $k \neq 3, m \in R$

13. Let $f : R \to R$ be defined as $f(x) = 2x - 1$ and $g : R - \{1\} \to R$ be defined as $g(x) = \frac{x - \frac{1}{2}}{x - 1}$. Then the composition function $f(g(x))$ is

(a) onto but not one-one

(b) neither one-one nor onto

(c) one-one but not onto

(d) both one-one and onto

14. Let p and q be two positive numbers such that $p + q = 2$ and $p^4 + q^4 = 272$. Then p and q are roots of the equation

(a) $x^2 - 2x + 16 = 0$ (b) $x^2 - 2x + 2 = 0$

(c) $x^2 - 2x + 136 = 0$ (d) $x^2 - 2x + 8 = 0$

15. An ordinary dice is rolled for a certain number of times. If the probability of getting an odd number 2 times is equal to the probability of getting an even number 3 times, then the probability of getting an odd number for odd number of times is

(a) $\frac{5}{16}$ (b) $\frac{3}{16}$ (c) $\frac{1}{2}$ (d) $\frac{1}{32}$

16. The equation of the plane passing through the point (1, 2, –3) and perpendicular to the planes $3x + y - 2z = 5$ and $2x - 5y - z = 7$, is

(a) $6x - 5y - 2z - 2 = 0$

(b) $6x - 5y + 2z + 10 = 0$

(c) $3x - 10y - 2z + 11 = 0$

(d) $11x + y + 17z + 38 = 0$

17. The locus of the mid-point of the line segment joining the focus of the parabola $y^2 = 4ax$ to a moving point of the parabola, is another parabola whose directrix is

(a) $x = \frac{a}{2}$ (b) $x = 0$

(c) $x = -\frac{a}{2}$ (d) $x = a$

18. The value of $-{}^{15}C_1 + 2\cdot{}^{15}C_2 - 3\cdot{}^{15}C_3 + \ldots\ldots - 15\cdot{}^{15}C_{15} + {}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + \ldots\ldots + {}^{14}C_{11}$ is

(a) $2^{13} - 14$ (b) 2^{14}

(c) $2^{16} - 1$ (d) $2^{13} - 13$

19. If the tangent to the curve $y = x^3$ at the point $P(t, t^3)$ meets the curve again at Q, then the ordinate of the point which divides PQ internally in the ratio 1 : 2 is

(a) $-t^3$ (b) $-2t^3$ (c) $2t^3$ (d) 0

20. Two vertical poles are 150 m apart and the height of one is three times that of the other. If from the middle point of the line joining their feet, an observer finds the angles of elevation of their tops to be complementary, then the height of the shorter pole (in meters) is

(a) 30 (b) 25 (c) $20\sqrt{3}$ (d) $25\sqrt{3}$

SECTION-B (NUMERICAL VALUE TYPE)

Attempt any 5 out of 10.

21. Let B_i $(i = 1, 2, 3)$ be three independent events in a sample space. The probability that only B_1 occur is α, only B_2 occurs is β and only B_3 occurs is γ. Let p be the probability that none of the events B_i occurs and these 4 probabilities satisfy the equations $(\alpha - 2\beta)\,p = \alpha\beta$ and $(\beta - 3\gamma)\,p = 2\beta\gamma$ (All the probabilities are assumed to lie in the interval (0, 1)). Then $\frac{P(B_1)}{P(B_3)}$ is equal to ______.

22. $\lim_{n\to\infty} \tan\left\{\sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{1+r+r^2}\right)\right\}$ is equal to ______.

23. If the least and the largest real values of α, for which the equation $z + \alpha|z - 1| + 2i = 0$ $(z \in C$ and $i = \sqrt{-1})$ has a solution, are p and q respectively; then $4(p^2 + q^2)$ is equal to ______.

24. If one the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord of another circle 'C', whose center is at (2, 1), then its radius is ______.

25. Let M be any 3×3 matrix with entries from the set $\{0, 1, 2\}$. The maximum number of such matrices, for which the sum of diagonal elements of M^TM is seven, is ______.

26. If $\int_{-a}^{a}(|x|+|x-2|)dx = 22, (a > 2)$ and $[x]$ denotes the greatest integer $\leq x$, then $\int_{a}^{-a}(x+[x])dx$ is equal to ______.

27. Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in R$. Suppose $Q = [q_{ij}]$ is a matrix satisfying $PQ = kI_3$ for some non-zero $k \in R$. If $q_{23} = -\frac{k}{8}$ and $|Q| = \frac{k^2}{2}$, then $\alpha^2 + k^2$ is equal to ______.

28. Let three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ be such that $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$, $\vec{a} \cdot \vec{c} = 7$ and $\vec{b}$ is perpendicular to $\vec{c}$, where $\vec{a} = -\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{k}$, then the value of $2|\vec{a} + \vec{b} + \vec{c}|^2$ is ______.

29. Let $A = \{n \in N : n \text{ is a 3-digit number}\}$
$B = \{9k + 2 : k \in N\}$
and $C = \{9k + l : k \in N\}$ for some $l (0 < l < 9)$
If the sum of all the elements of the set $A \cap (B \cup C)$ is 274×400, then l is equal to ______.

30. The minimum value of α for which the equation $\frac{4}{\sin x} + \frac{1}{1 - \sin x} = \alpha$ has at least one solution in $\left(0, \frac{\pi}{2}\right)$ is ______.

SOLUTIONS

1. **(c) :** Let the line be $y = mx + c$ ...(i)

$\therefore$ x-intercept is $-\frac{c}{m}$ and y-intercept is c.

A.M. of reciprocals of the intercepts is given by

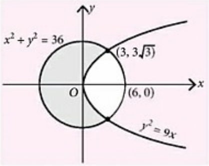
$$\frac{-\frac{m}{c} + \frac{1}{c}}{2} = \frac{1}{4} \Rightarrow 2(1 - m) = c$$

From (i), $y = mx + 2(1 - m)$

$\Rightarrow (y - 2) - m(x - 2) = 0$, which passes through $(2, 2)$.

2. **(d) :** Points of intersection of given curves are $(3, \pm 3\sqrt{3})$.

$\therefore$ Required area $= \pi \times (6)^2 - 2\int_0^3 \sqrt{9x}\,dx - 2\int_3^6 \sqrt{36 - x^2}\,dx$



$$= 36\pi - 6\left[\frac{x^{3/2}}{3/2}\right]_0^3 - 2\left[\frac{x}{2}\sqrt{36 - x^2} + 18\sin^{-1}\frac{x}{6}\right]_3^6$$

$$= 36\pi - 12\sqrt{3} - 2\left(9\pi - 3\pi - \frac{9\sqrt{3}}{2}\right)$$

$$= \left(24\pi - 3\sqrt{3}\right) \text{ sq. units}$$

3. **(b) :** $e^{(\cos^2 x + \cos^4 x + \cos^6 x + ... \infty)\log_e 2}$

$$= 2^{\cos^2 x + \cos^4 x + ... \infty} = 2^{\cos^2 x\left(\frac{1}{1 - \cos^2 x}\right)} = 2^{\cot^2 x}$$

Now, $t^2 - 9t + 8 = 0 \Rightarrow t = 1, 8$

$\Rightarrow 2^{\cot 2x} = 1, 8 \Rightarrow \cot^2 x = 0, 3$

Since, $0 < x < \frac{\pi}{2} \Rightarrow \cot x = \sqrt{3}$

$\therefore \frac{2\sin x}{\sin x + \sqrt{3}\cos x} = \frac{2}{1 + \sqrt{3}\cot x} = \frac{2}{4} = \frac{1}{2}$

4. **(c) :**

Indians	Foreigners
2	4
3	6
4	8

$\therefore$ Total number of ways

$= {}^6C_2 \times {}^8C_4 + {}^6C_3 \times {}^8C_6 + {}^6C_4 \times {}^8C_8$

$= 15 \times 70 + 20 \times 28 + 15 \times 1$

$= 1050 + 560 + 15 = 1625$

5. **(a) :** We have, $\frac{dP}{dt} = 0.5P - 450$

$$\Rightarrow \frac{dP(t)}{dt} = \frac{P(t) - 900}{2} \Rightarrow \int_0^t \frac{dP(t)}{P(t) - 900} = \int_0^t \frac{dt}{2}$$

$$\Rightarrow \left[\log|P(t) - 900|\right]_0^t = \left[\frac{t}{2}\right]_0^t$$

$$\Rightarrow \log|P(t) - 900| - \log|P(0) - 900| = \frac{t}{2}$$

$$\Rightarrow \log|P(t) - 900| - \log|50| = \frac{t}{2} \quad [\because P(0) = 850]$$

When $P(t) = 0$, then

$$\log_e\left|\frac{900}{50}\right| = \frac{t}{2} \Rightarrow t = 2\log_e 18$$

6. (c) : Let $I=\int \frac{\cos x-\sin x}{\sqrt{8-\sin 2x}}\,dx$

$$=\int \frac{\cos x-\sin x}{\sqrt{9-(\sin x+\cos x)^2}}\,dx$$

Put $\sin x + \cos x = t \Rightarrow (\cos x - \sin x)dx = dt$

$$\therefore\ I=\int \frac{dt}{\sqrt{9-t^2}}=\sin^{-1}\frac{t}{3}+c=\sin^{-1}\left(\frac{\sin x+\cos x}{3}\right)+c$$

So, $a = 1, b = 3$

7. (a) : We have, $f(x)=\frac{4x^3-3x^2}{6}-2\sin x+(2x-1)\cos x$

$\Rightarrow f'(x) = (2x^2 - x) - 2\cos x + 2\cos x - \sin x(2x - 1)$

$\quad = (2x - 1)(x - \sin x)$

For $x > 0$, $x - \sin x > 0$ and $x < 0$, $x - \sin x < 0$

For $x\in\left[\frac{1}{2},\infty\right)$, $f'(x)\geq 0$ and for $x\in\left(0,\frac{1}{2}\right]$, $f'(x)\leq 0$

$\therefore\ f(x)$ increases in $\left[\frac{1}{2},\infty\right)$.

8. (b) : $(A\wedge(A\to B))\to B\equiv(A\wedge(\sim A\vee B))\to B$

$\equiv((A\wedge\sim A)\vee(A\wedge B)\to B\equiv(A\wedge B)\to B$

$\equiv(\sim A\vee\sim B)\vee B\equiv T$

9. (a) : $\lim\limits_{x\to 0}\frac{\int_0^{x^2}\sin\sqrt{t}\,dt}{x^3}\quad\left(\frac{0}{0}\text{ form}\right)$

$=\lim\limits_{x\to 0}\frac{(\sin x)2x}{3x^2}$ [Applying L' Hospital's Rule]

$=\lim\limits_{x\to 0}\left(\frac{\sin x}{x}\right)\times\frac{2}{3}=\frac{2}{3}$

10. (a) : For $x = n$, $n \in Z$

L.H.L. $=\lim\limits_{x\to n^-}f(x)=\lim\limits_{x\to n^-}[x-1]\cos\left(\frac{2x-1}{2}\right)\pi=0$

R.H.L. $=\lim\limits_{x\to n^+}f(x)=\lim\limits_{x\to n^+}[x-1]\cos\left(\frac{2x-1}{2}\right)\pi=0\ =0$

Also, $f(n) = 0$

$\therefore$ L.H.L. = R.H.L. = $f(n)$

So, $f(x)$ is continuous for every real x.

11. (a) : Let $\frac{x-3}{1}=\frac{y-4}{2}=\frac{z-5}{2}=t$ (say)

$\therefore\ x = 3 + t, y = 2t + 4, z = 2t + 5$, which lie on the plane $x + y + z = 17$

$\therefore\ 3 + t + 2t + 4 + 2t + 5 = 17 \Rightarrow 5t = 5 \Rightarrow t = 1$

$\therefore$ Point of intersection is (4, 6, 7)

Hence, distance between (1, 1, 9) and (4, 6, 7)

$=\sqrt{(4-1)^2+(6-1)^2+(7-9)^2}\ =\sqrt{9+25+4}=\sqrt{38}$

12. (a) : $D=\begin{vmatrix}3&-2&-k\\2&-4&-2\\1&2&-1\end{vmatrix}=0$

$\Rightarrow 24 - 2(0) - k(8) = 0 \Rightarrow k = 3$

At $k = 3$,

$$D_1=\begin{vmatrix}10&-2&-3\\6&-4&-2\\5m&2&-1\end{vmatrix}=0$$

$= 10(8) - 2(-10m + 6) - 3(12 + 20m) = 8(4 - 5m)$

For the given system to be inconsistent, $D_1 \neq 0$

$\Rightarrow\ 8(4-5m)\neq 0\ \Rightarrow\ m\neq\frac{4}{5}$

$\therefore$ System of linear equations is inconsistent, if $k = 3$ and $m\neq\frac{4}{5}$.

13. (c) : $f(g(x))=2g(x)-1=2\left(\frac{2x-1}{2(x-1)}\right)-1$

$$=\frac{x}{x-1}=1+\frac{1}{x-1}$$

Range of $f(g(x)) = R - \{1\}$

Thus, range of $f(g(x))$ is not co-domain of $f(g(x))$.

$\therefore\ f(g(x))$ is not onto.

But $f(g(x))$ is one-one.

So, $f(g(x))$ is one-one but not onto.

14. (a) : Consider, $p^4 + q^4 = 272$

$\Rightarrow (p^2 + q^2)^2 - 2p^2q^2 = 272$

$\Rightarrow ((p + q)^2 - 2pq)^2 - 2p^2q^2 = 272$

$\Rightarrow (p + q)^4 + 4p^2q^2 - 4pq(p + q)^2 - 2p^2q^2 = 272$

$\Rightarrow 16 - 16pq + 2p^2q^2 = 272$ [Using $p + q = 2$]

$\Rightarrow (pq)^2 - 8(pq) - 128 = 0$

$\therefore\ pq=\frac{8\pm 24}{2}=16,-8\Rightarrow pq=16$

Thus, p, q are the roots of the equation, $x^2 - 2x + 16 = 0$.

15. (c) : P(getting an odd number twice)

= P(getting an even number thrice)

$\Rightarrow\ {}^nC_2\left(\frac{1}{2}\right)^n={}^nC_3\left(\frac{1}{2}\right)^n\ \Rightarrow\ {}^nC_2={}^nC_3\Rightarrow n=2+3$

$\Rightarrow\ n = 5$

Probability of getting an odd number for odd number of times $={}^5C_1\left(\frac{1}{2}\right)^5+{}^5C_3\left(\frac{1}{2}\right)^5+{}^5C_5\left(\frac{1}{2}\right)^5=\frac{1}{2^5}(5+10+1)$

$=\frac{16}{2^5}=\frac{1}{2}$

16. (d) : Normal vector of required plane is

$$\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\3&1&-2\\2&-5&-1\end{vmatrix}=-11\hat{i}-\hat{j}-17\hat{k}$$

Also, plane passes through (1, 2, –3).
So, equation of required plane is

$$11(x-1)+1(y-2)+17(z+3)=0$$

$$\Rightarrow 11x+y+17z+38=0$$

17. (b) : Here, $h=\dfrac{at^2+a}{2}$ and $k=\dfrac{2at+0}{2}$

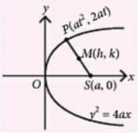
$$\Rightarrow t^2=\frac{2h-a}{a} \text{ and } t=\frac{k}{a}$$

$$\therefore \frac{k^2}{a^2}=\frac{2h-a}{a} \Rightarrow k^2=a(2h-a)$$

Locus of (h, k) is $y^2=a(2x-a)$

$$\Rightarrow y^2=2a\left(x-\frac{a}{2}\right), \text{ which is a parabola.}$$

$$\therefore \text{ Its directrix is } x-\frac{a}{2}=-\frac{a}{2} \Rightarrow x=0$$



18. (a) : We have

$$({-}^{15}C_1+2\cdot{}^{15}C_2-3\cdot{}^{15}C_3+\,.....\,-15\cdot{}^{15}C_{15}) + ({}^{14}C_1+{}^{14}C_3+\,.......\,{}^{14}C_{11})$$

$$=\sum_{r=1}^{15}(-1)^r r.{}^{15}C_r+({}^{14}C_1+{}^{14}C_3+....+{}^{14}C_{11}+{}^{14}C_{13})-{}^{14}C_{13}$$

$$=\sum_{r=1}^{15}(-1)^r r.\frac{15}{2}\,{}^{14}C_{r-1}+2^{13}-14$$

$$=15({-}^{14}C_0+{}^{14}C_1-\,.....\,-{}^{14}C_{14})+2^{13}-14$$

$$=15(0)+2^{13}-14=2^{13}-14$$

19. (b) : We have, $y=x^3$ ...(i)

Slope of tangent at $P(t, t^3)$ is

$$\left[\frac{dy}{dx}\right]_{(t,\,t^3)}=(3x^2)_{x=t}=3t^2$$

So, equation of tangent at $P(t, t^3)$ is given by

$$y-t^3=3t^2(x-t) \qquad ...(ii)$$

From (i) and (ii), we get $x^3-t^3=3t^2(x-t)$

$$\Rightarrow (x-t)(x^2+xt+t^2)=3t^2(x-t)$$

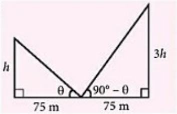
$$\Rightarrow x^2+xt+t^2=3t^2 \qquad [\because\ x\neq t]$$

$$\Rightarrow x^2+xt-2t^2=0 \Rightarrow (x-t)(x+2t)=0$$

$$\therefore x=-2t \text{ and } y=-8t^3$$

$$\therefore \text{ Ordinate of required point is, } \frac{2t^3+(-8t^3)}{2+1}=-2t^3$$

20. (d) :



Here, $\tan\theta=\dfrac{h}{75}=\dfrac{75}{3h} \Rightarrow h^2=\dfrac{(75)^2}{3} \Rightarrow h=25\sqrt{3}$ m

21. (6) : Let $P(B_1)=p_1, P(B_2)=p_2, P(B_3)=p_3$

Given, $p_1(1-p_2)(1-p_3)=\alpha$ ...(i)

$p_2(1-p_1)(1-p_3)=\beta$ ...(ii)

$p_3(1-p_1)(1-p_2)=\gamma$ ...(iii)

and $(1-p_1)(1-p_2)(1-p_3)=p$ ...(iv)

$$\Rightarrow \frac{p_1}{1-p_1}=\frac{\alpha}{p}, \frac{p_2}{1-p_2}=\frac{\beta}{p} \text{ and } \frac{p_3}{1-p_3}=\frac{\gamma}{p}$$

$$\text{Also, } \beta=\frac{\alpha p}{\alpha+2p}=\frac{3\gamma p}{p-2\gamma}$$

$$\Rightarrow \alpha p-2\alpha\gamma=3\alpha\gamma+6p\gamma \Rightarrow \alpha p-6p\gamma=5\alpha\gamma$$

$$\Rightarrow \frac{p_1}{1-p_1}-\frac{6p_3}{1-p_3}=\frac{5p_1p_3}{(1-p_1)(1-p_3)} \Rightarrow p_1-6p_3=0$$

$$\Rightarrow \frac{p_1}{p_3}=6 \text{ i.e., } \frac{P(B_1)}{P(B_3)}=6$$

22. (1) : $\displaystyle\lim_{n\to\infty}\tan\left(\sum_{r=1}^{n}\tan^{-1}\left(\frac{1}{1+r(r+1)}\right)\right)$

$$=\lim_{n\to\infty}\tan\left(\sum_{r=1}^{n}\tan^{-1}\left(\frac{r+1-r}{1+r(r+1)}\right)\right)$$

$$=\tan\left(\lim_{n\to\infty}\sum_{r=1}^{n}[\tan^{-1}(r+1)-\tan^{-1}(r)]\right)$$

$$=\tan\left(\lim_{n\to\infty}\left(\tan^{-1}(n+1)-\tan^{-1}1\right)\right)$$

$$=\tan(\tan^{-1}(\infty)-\frac{\pi}{4}=\tan\left(\frac{\pi}{2}-\frac{\pi}{4}\right)=\tan\left(\frac{\pi}{4}\right)=1$$

23. (10) : The given equation is $z+\alpha|z-1|+2i=0$

Putting $z=x+iy$, we get

$$x+iy+\alpha|x+iy-1|+2i=0$$

$$\Rightarrow x+\alpha\sqrt{(x-1)^2+y^2}+i(y+2)=0+0i$$

$$\Rightarrow y+2=0 \text{ and } x+\alpha\sqrt{(x-1)^2+y^2}=0$$

$$\Rightarrow y=-2 \text{ and } \alpha^2=\frac{x^2}{x^2-2x+5}$$

$$\text{Now, } \frac{x^2}{x^2-2x+5}\in\left[0,\frac{5}{4}\right]$$

$$\therefore \alpha^2\in\left[0,\frac{5}{4}\right] \Rightarrow \alpha\in\left[-\frac{\sqrt{5}}{2},\frac{\sqrt{5}}{2}\right]$$

$$\therefore p=-\frac{\sqrt{5}}{2};\ q=\frac{\sqrt{5}}{2}$$

$$\Rightarrow 4(p^2+q^2)=4\left(\frac{5}{4}+\frac{5}{4}\right)=10$$

24. (3) : The given equation of circle is $x^2+y^2-2x-6y+6=0$, whose center is (1, 3) and radius is

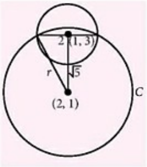
$$\sqrt{(1)^2+(3)^2-6}=2$$

$\therefore$ Distance between (1, 3) and (2, 1) is

$$\sqrt{(2-1)^2+(1-3)^2}=\sqrt{5}.$$

$\therefore \quad (\sqrt{5})^2+(2)^2=r^2 \Rightarrow r^2=9$

$\Rightarrow r=3$



25. (540) : Let $M=\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix} \Rightarrow M^T=\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

$$\therefore \quad M^TM=\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

Since, sum of diagonal elements of $M^TM=7$

$\Rightarrow a^2+b^2+c^2+d^2+e^2+f^2+g^2+h^2+i^2=7$

Case-I : Seven (1's) and two (0's)

So, number of possible matrices $=\dfrac{9!}{7!2!}=36$

Case-II : One (2's), three (1's) and five (0's)

So, number of possible matrices $=\dfrac{9!}{5!3!}=504$

$\therefore$ Total number of possible matrices = 504 + 36 = 540

26. (3) : $\displaystyle\int_{-a}^{0}(-2x+2)dx+\int_{0}^{2}(x+2-x)dx+\int_{2}^{a}(2x-2)dx=22$

$\Rightarrow \left[x^2-2x\right]_0^{-a}+[2x]_0^2+\left[x^2-2x\right]_2^a=22$

$\Rightarrow a^2+2a+4+a^2-2a-(4-4)=22$

$\Rightarrow 2a^2=18 \Rightarrow a=3$

$\therefore \displaystyle\int_{3}^{-3}(x+[x])dx=-\int_{-3}^{3}[x]dx=-(-3-2-1+1+2)=3$

27. (17) : Given $PQ=kI_3 \Rightarrow |PQ|=k^3|I_3|$

$\Rightarrow |P|\cdot|Q|=k^3 \Rightarrow |P|\dfrac{k^2}{2}=k^3$

$\Rightarrow |P|=2k\neq 0 \Rightarrow P$ is an invertible matrix.

$\therefore \quad Q=kP^{-1}I=\dfrac{k\,\mathrm{adj}P}{|P|}=\dfrac{k}{2k}\mathrm{adj}P \Rightarrow Q=\dfrac{\mathrm{adj}P}{2}$

Now, $q_{23}=-\dfrac{k}{8} \Rightarrow \dfrac{-(3\alpha+4)}{2}=-\dfrac{k}{8}$ ...(i)

Now, $|P|=2k \Rightarrow 12\alpha+20=2k \Rightarrow k=10+6\alpha$

From (i), $\dfrac{-(3\alpha+4)}{2}=-\dfrac{(10+6\alpha)}{8} \Rightarrow \alpha=-1$

and $k=10+6(-1)=4$

$\therefore \quad \alpha^2+k^2=(-1)^2+4^2=17$

28. (75) : Let $\vec{c}=\lambda(\vec{b}\times(\vec{a}\times\vec{b}))=\lambda((\vec{b}\cdot\vec{b})\vec{a}-(\vec{b}\cdot\vec{a})\vec{b})$

$=\lambda(5(-\hat{i}+\hat{j}+\hat{k})+2\hat{i}+\hat{k})=\lambda(-3\hat{i}+5\hat{j}+6\hat{k})$

Also, $\vec{a}\cdot\vec{c}=7 \Rightarrow 3\lambda+5\lambda+6\lambda=7 \Rightarrow 14\lambda=7$

$\Rightarrow \lambda=\dfrac{1}{2}$

Now, $\vec{a}+\vec{b}+\vec{c}=\left(\dfrac{-3}{2}-1+2\right)\hat{i}+\left(\dfrac{5}{2}+1\right)\hat{j}+(3+1+1)\hat{k}$

$=-\dfrac{1}{2}\hat{i}+\dfrac{7}{2}\hat{j}+5\hat{k}$

$\therefore \quad 2|\vec{a}+\vec{b}+\vec{c}|^2=2\left(\dfrac{1}{4}+\dfrac{49}{4}+25\right)=2\left(\dfrac{150}{4}\right)=75$

29. (5) : B and C will contain three digit numbers of the form $9k+2$ and $9k+l$ respectively. We need to find sum of all elements in the set $B\cup C$.

Now, $S(B\cup C)=S(B)+S(C)-S(B\cap C)$,

where $S(k)$ denotes sum of elements of set k.

Also, $B=\{101, 110, ... 992\}$

$\therefore \quad S(B)=\dfrac{100}{2}(101+992)=54650$

Case-I : If $l=2$, then $B\cap C=B$

$\therefore \quad S(B\cup C)=S(B)$, which is not possible as given sum is $274\times 400=109600$

Case-II : If $l\neq 2$, then $B\cap C=\phi$

$\therefore \quad S(B\cup C)=S(B)+S(C)=400\times 274$

$\Rightarrow \displaystyle 54650+\sum_{k=11}^{110}(9k+l)=109600$

$\Rightarrow \displaystyle 9\sum_{k=11}^{110}k+\sum_{k=11}^{110}l=54950$

$\Rightarrow 9\left(\dfrac{100}{2}(11+110)\right)+l(100)=54950$

$\Rightarrow 54450+100l=54950 \Rightarrow l=5$

30. (9) : Let $y=\dfrac{4}{\sin x}+\dfrac{1}{1-\sin x}=\dfrac{4-3\sin x}{\sin x(1-\sin x)}$

Let $\sin x=t$, where $t\in(0, 1)$

$\therefore \quad y=\dfrac{4-3t}{t-t^2}\ ;\ \dfrac{dy}{dt}=\dfrac{(t-t^2)(-3)-(4-3t)(1-2t)}{(t-t^2)^2}=0$

$\Rightarrow 3t^2-3t-(4-11t+6t^2)=0$

$\Rightarrow 3t^2-8t+4=0 \Rightarrow (3t-2)(t-2)=0$

$\Rightarrow t=\dfrac{2}{3}\quad [\because t\in(0,1)] \Rightarrow \sin x=\dfrac{2}{3}$

$\therefore \quad y_{\min}=\dfrac{4}{2/3}+\dfrac{1}{1-2/3}=9$

$\because \quad y_{\max}\to\infty$ and y is continuous function in $\left(0, \dfrac{\pi}{2}\right)$.

$\therefore \quad \alpha_{\min}=9$

◆◆

Revision tips to help you get exam-ready

Amarnath Anand

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HOD of Mathematics Department
Author of Thomas Calculus for JEE and Problem and Solution in Algebra for IIT JEE
Mentored AIR 1 and AIR 3

> **"करत करत अभ्यास के जड़मति होत सुजान**
> **रसरी आवत जात ते सिल पर पड़त निशान"**

I read this quote during my childhood. It says that even if you have zero knowledge, with practice you can excel in anything and I strongly believe that. Revision is the key to perfection. Even if you do not have confidence in a particular topic, if you revise it over and over again, you will get better at it. However, the problem is that Learners are not very enthusiastic about revision. That is, they lack motivation and they alienate it. I am sure you have experienced it too.

However, there is a trick or a method to tackle this. Firstly, you need to understand that revision is a part of your studies. It is an integral part of your preparation journey and you need to prioritise it. What you can do is that, while revising, make use of the asterisk (*). Mark one asterisk against the questions you solve in one attempt, two against those that need two attempts and so on. You can also use 'T' against doubts or questions that you have asked your Educator. Such tagging will simplify your revision process. While revising, ensure that you focus or pay attention to the questions that have more than one asterisk marks or are marked with a 'T'.

Also, while revising Mathematics, make one copy that encompasses the formulas of every chapter. You can also use one single A-4 size paper to write all questions or class illustrations of a particular chapter. This way, you do not have to go back to the book every time, you can simply refer to the A-4 size paper during revision. This will help you save time and enhance your preparation.

It is also advisable to make a strict revision schedule. The schedule can be planned 15-days at a time. Decided on which topics you will revise and study for the next 15 days and ensure that you dedicate atleast 1-2 hours everyday for revision.

All the best. I hope these tips will help you. Remember, there is nothing unattainable to the ones who have the courage to work hard.

॥ अप्राप्यं नाम नेहास्ति धीरस्य व्यवसायिनः ॥

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JEE MAIN

Exam Dates 2021

27th to 30th April

24th to 28th May

Chapters covered : Complex Numbers, Quadratic Equations, Permutations and Combinations, Properties of Triangles, Straight Lines and Circles

SECTION-A (MULTIPLE CHOICE QUESTIONS)

1. If $\alpha = \cos\frac{8\pi}{11} + i\sin\frac{8\pi}{11}$, then $\text{Re}(\alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5) =$

(a) 0 (b) $-\frac{1}{2}$

(c) $\frac{1}{2}$ (d) none of these

2. If the equations $ax^2 + bx + c = 0$ and $cx^2 + bx + a = 0$ have one root in common, then the value of $a^3 + b^3 + c^3 =$

(a) $3a(a + b + c)$

(b) $a^2 + b^2 + c^2 - ab - bc - ca$

(c) $3(ab + bc + ca)$

(d) $3abc$

3. A variable straight line passes through the intersection of the lines $x + 2y = 1$ and $2x - y = 1$ meet the co-ordinate axes A and B , then the locus of middle point of AB is

(a) $x - 3y = 10xy$ (b) $x + 3y = 10xy$

(c) $3x + y = 10xy$ (d) none of these

4. If $p, q, r \in R$, then the roots of the equation $p(x^2 - z^2) - (q - r)xz = 0$ are

(a) imaginary (b) real but not equal

(c) real and equal (d) none of these

5. Let $y = \frac{mx^2 + 3x - 4}{m + 3x - 4x^2}$, the values of m for which $y \in R, \forall x \in R$ is

(a) $-7 \le m \le -1$ (b) $m \ge -4$

(c) $1 \le m \le 7$ (d) none of these

6. If R_1 and R_2 be the perpendicular drawn from the origin to the line $\frac{x}{\cos\theta} + \frac{y}{\sin\theta} = a$ and $\frac{x}{\sec\theta} - \frac{y}{\operatorname{cosec}\theta} = \frac{a}{\sec 2\theta}$ respectively, then the value of the expression $4R_1^2 + R_2^2$ equals

(a) $4a^2$ (b) $3a^2$ (c) $2a^2$ (d) a^2

7. The lines joining the origin to the point of intersection of $3x - 4y = 56$ and the curve $x^2 + y^2 - 24x + 10y = 0$ is

(a) coincident lines

(b) lines are at 90°

(c) equal in length

(d) makes equal angles with y-axis

8. C_1 and C_2 are two circles touching each other and the coordinate axes. If C_1 is smaller than C_2 and its radius is 2 units, then radius of C_2 is

(a) $6 + 4\sqrt{2}$ (b) $2 + 2\sqrt{2}$

(c) $3 + 2\sqrt{2}$ (d) none of these.

9. From a point $A(1, 1)$ on the circle $x^2 + y^2 - 4x - 4y + 6 = 0$, two chords AB and AC each of length 2 units are drawn. The equation of chord BC is

(a) $4x + 3y - 12 = 0$ (b) $x + y = 4$

(c) $3x + 4y = 4$ (d) $x + y = 6$

10. If the lines $3x - 4y - 7 = 0$ and $2x - 3y - 5 = 0$ are diameters of a circle of area 49π square units, then the equation of the circle is

(a) $x^2 + y^2 + 2x - 2y - 62 = 0$

(b) $x^2 + y^2 - 2x + 2y - 62 = 0$

(c) $x^2 + y^2 - 2x + 2y - 47 = 0$

(d) $x^2 + y^2 + 2x - 2y - 47 = 0$

11. The value of ${}^{35}C_8 + \sum_{r=1}^{7} {}^{42-r}C_7 + \sum_{s=1}^{5} {}^{47-s}C_{40-s}$ is

(a) ${}^{47}C_7$ (b) ${}^{46}C_8$ (c) ${}^{47}C_6$ (d) ${}^{47}C_8$

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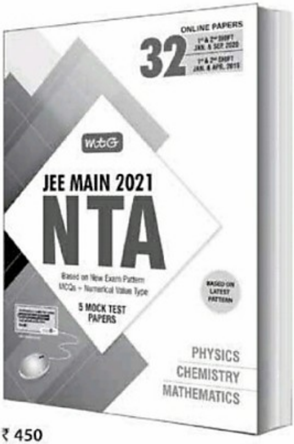
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12. Eighteen guests have to be seated, half on each side of a long table. Four particular guest desires to sit on one particular side and three others on the other side. The number of ways in which the seating arrangement can be made, is

(a) $9! \times 9!$ (b) $^{11}C_5 \times 9! \times 9!$

(c) $\frac{11!}{5!} \times 9! \times 9!$ (d) $^{11}C_5$

13. Let L_1 be a straight line passing through the origin and L_2 be the straight line $x + y = 1$. If the intercepts made by the circle $x^2 + y^2 - x + 3y = 0$ on L_1 and L_2 are equal, then which of the following equations can represent L_1 ?

(a) $x + y = 0, x - 7y = 0$ (b) $x - y = 0, x + 7y = 0$

(c) $7x + y = 0$ (d) $x - 7y = 0$

14. The angle between the tangents from (α, β) to the circle $x^2 + y^2 = a^2$ is

(a) $\tan^{-1}\left(\frac{a}{\sqrt{s_1}}\right)$ (b) $2\tan^{-1}\left(\frac{a}{\sqrt{s_1}}\right)$

(c) $2\tan^{-1}\left(\frac{\sqrt{s_1}}{a}\right)$ (d) none of these.

Where $s_1 = \alpha^2 + \beta^2 - a^2$.

15. Length of two sides of a triangle are given by the roots of the equation $x^2 - 2\sqrt{3}x + 2 = 0$. The angle between the sides is $\frac{\pi}{3}$. The perimeter of the triangle is

(a) $6 + \sqrt{3}$ (b) $2\sqrt{3} + \sqrt{6}$

(c) $2\sqrt{3} + \sqrt{10}$ (d) none of these.

16. In ΔABC, AD and BE are the medians drawn through the angular points A and B respectively. $\angle DAB = 2\angle ABE = 36°$ and $AD = 6$ units, then circumradius of the triangle is equal to

(a) $(3-\sqrt{5})\text{cosec}C$ (b) $(3+\sqrt{5})\text{cosec}C$

(c) $2(3-\sqrt{5})\text{cosec}C$ (d) $2(3+\sqrt{5})\text{cosec}C$

17. If the inradius of a circle inscribed in an isosceles triangle whose one angle is $\frac{2\pi}{3}$ is $\sqrt{3}$, then the area of the triangle in square units is

(a) $7 + 12\sqrt{3}$ (b) $12 - 7\sqrt{3}$

(c) $12 + 7\sqrt{3}$ (d) 4π

18. The locus of $z = a + ib$ which satisfy the inequality $\log_{\frac{1}{2}}|z-1| > \log_{\frac{1}{2}}|z-i|$ is given by

(a) $a + b < 0$ (b) $a - b > 0$

(c) $a - b < 0$ (d) $a + b > 0$

19. Let z and ω are two non-zero complex numbers such that $|z| = |\omega|$ and $\arg(z) + \arg(\omega) = \pi$, then z equals

(a) $\bar{\omega}$ (b) ω (c) $-\bar{\omega}$ (d) $-\omega$

20. If the roots of the equation $x^2 - \omega x + v = 0$ differ by 2, then

(a) $v^2 = 4(1 + \omega)$ (b) $v^2 = 1(1 - \omega)$

(c) $\omega^2 = 4(1 + v)$ (d) $\omega^2 = 4(1 - v)$

SECTION-B (NUMERICAL VALUE TYPE)

Attempt any 5 out of 10.

21. If the area of the triangle on the complex plane formed by the points z, iz, $z + iz$ is 200 square units, then $|z| =$ _______.

22. Two parallel chords are drawn on the same side of the centre of a circle of radius 12 units. It is found that they subtend 72° and 144° at the centre of the circle. The perpendicular distance (in units) between the chords is _______.

23. If a, b, c are positive integer and $f(x) = x^{6a} + x^{6b+1} + x^{6c+2}$, then $f(\omega) =$ _______.

24. The number of triangles whose vertices are at the vertices of an octagon but none of whose sides happen to come from the sides of the octagon is _______.

25. If the image of the point (3, 8) in the line $px + 3y - 7 = 0$ is $(-1, -4)$, then value of p is _______.

26. The letters of the word "RANDOM" are written in all possible orders and these words are written out as in a dictionary, then the rank of the word "RANDOM" is _______.

27. There are five balls of different colours and five boxes of colours same as those of the balls. The number of ways in which the balls, one each in a box, could be placed such that exactly one ball goes to a box of its own colour, is _______.

28. If the nearest point on the line $3x + 4y = 25$ from the origin is (a, b), then $b - a$ equals _______.

29. If (2, 4) is a point interior to the circle $x^2 + y^2 - 6x - 10y + \lambda = 0$ and the circle does not cut the axes at any point, then, $\lambda \in (a, b)$. The value of $a + b$ equals _______.

30. The sum of all five digit numbers that can be formed using the digits 1, 2, 3, 4, 5, when repetition of digits is not allowed, is _______.

SOLUTIONS

1. **(b) :** We know that $\text{Re}(z) = \dfrac{z+\bar{z}}{2}$

$\therefore$ $\text{Re}(\alpha+\alpha^2+\alpha^3+\alpha^4+\alpha^5)$

$$=\frac{(\alpha+\alpha^2+\alpha^3+\alpha^4+\alpha^5)+\overline{(\alpha+\alpha^2+\alpha^3+\alpha^4+\alpha^5)}}{2}$$

$$=\frac{1}{2}\left(\alpha+\alpha^2+\alpha^3+\alpha^4+\alpha^5+\frac{1}{\alpha}+\frac{1}{\alpha^2}+\frac{1}{\alpha^3}+\frac{1}{\alpha^4}+\frac{1}{\alpha^5}\right)$$

$$=\frac{1}{2\alpha^5}(1+\alpha+\alpha^2+\alpha^3+\ldots+\alpha^{10}-\alpha^5)$$

$$=\frac{1}{2\alpha^5}\left[\frac{1-\alpha^{11}}{1-\alpha}-\alpha^5\right]=\frac{1}{2\alpha^5}[0-\alpha^5]=-\frac{1}{2}$$

$$\left[\because\ \alpha=\cos\frac{8\pi}{11}+i\sin\frac{8\pi}{11}\Rightarrow\alpha^{11}=\cos 8\pi+i\sin 8\pi=1\right]$$

2. **(d) :** Let the common root be α.

$\therefore$ $a\alpha^2+b\alpha+c=0$ and $c\alpha^2+b\alpha+a=0$

$$\Rightarrow\ \frac{\alpha^2}{ba-bc}=\frac{\alpha}{c^2-a^2}=\frac{1}{ab-cb}$$

$\therefore$ $(c^2-a^2)^2=(ba-bc)(ba-bc)$

$\Rightarrow$ $(c+a)^2(c-a)^2=(ba-bc)^2$

$\Rightarrow$ $(c+a)^2(c-a)^2=b^2(c-a)^2$

$\Rightarrow$ $(c-a)^2\{(c+a)^2-b^2\}=0$

$\Rightarrow$ $c=a$ or $(c+a)^2-b^2=0$

$\Rightarrow$ $c=a$ or $(a+b+c)\,(a-b+c)=0$

$\Rightarrow$ $c=a$ or $(a+b+c)=0$ or $(a-b+c)=0$

From $a+b+c=0$, we get $a+b=-c$

$(a+b)^3=-c^3\Rightarrow a^3+b^3+3ab\,(a+b)=-c^3$

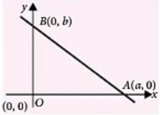
$\Rightarrow$ $a^3+b^3+3ab(-c)=-c^3\Rightarrow a^3+b^3+c^3=3abc$

3. **(b) :** The equation of the straight line passing through the point of intersection of the lines $x+2y=1$ and $2x-y=1$ is

$(x+2y-1)+\lambda(2x-y-1)=0$

$\Rightarrow$ $(1+2\lambda)x+(2-\lambda)y=(1+\lambda)$

$$\Rightarrow\ \frac{x}{\frac{1+\lambda}{1+2\lambda}}+\frac{y}{\frac{1+\lambda}{2-\lambda}}=1$$



$$\therefore\ A\equiv\left(\frac{1+\lambda}{1+2\lambda},0\right)$$

and $B\equiv\left(0,\dfrac{1+\lambda}{2-\lambda}\right)$

Let the mid-point of AB be (h, k)

$$\therefore\ h=\frac{1}{2}\left(\frac{1+\lambda}{1+2\lambda}\right)\text{ and }k=\frac{1}{2}\left(\frac{1+\lambda}{2-\lambda}\right)$$

Eliminating λ from h and k, we get $h+3k=10hk$.

Therefore the required locus is $x+3y=10xy$.

4. **(b) :** The given equation is $p(x^2-z^2)-(q-r)xz=0$

$$\Rightarrow\ px^2-(q-r)zx-pz^2=0\Rightarrow p\left(\frac{x}{z}\right)^2-(q-r)\frac{x}{z}-p=0$$

Now, $D=(q-r)^2+4p^2>0$. Thus the roots are real and distinct.

5. **(c) :** The given equation is $y=\dfrac{mx^2+3x-4}{m+3x-4x^2}$

$\Rightarrow$ $mx^2+3x-4=y(m+3x-4x^2)$

$\Rightarrow$ $(4y+m)x^2+3(1-y)x-(4+my)=0$

Since , $x, y\in R$, $\therefore$ $D\geq 0$ gives

$9(1-y)^2+4(4y+m)(4+my)\geq 0$

$\Rightarrow$ $(9+16m)y^2+(4m^2+46)y+(9+16m)\geq 0$

[If $ax^2+bx+c\geq 0$, then $a>0$ and $D\leq 0$]

$\therefore$ $9+16m>0$ and $4(2m^2+23)^2-4(9+16m)^2\leq 0$

$\Rightarrow$ $m>-\dfrac{9}{16}$ and $(2m^2+23)^2-(9+16m)^2\leq 0$

$\Rightarrow$ $(2m^2+23+9+16m)(2m^2+23-9-16m)\leq 0$

$\Rightarrow$ $2(m^2+8m+16)\times 2(m^2-8m+7)\leq 0$

$\Rightarrow$ $(m+4)^2(m-1)(m-7)\leq 0$

$\Rightarrow$ $1\leq m\leq 7$ as $(m+4)^2\geq 0$

6. **(d) :** The given equations of the straight line are

$$\frac{x}{\cos\theta}+\frac{y}{\sin\theta}=a\text{ and }\frac{x}{\sec\theta}-\frac{y}{\text{cosec}\,\theta}=\frac{a}{\sec 2\theta}$$

$$\therefore\ R_1=\frac{-a}{\sqrt{\sec^2\theta+\text{cosec}^2\theta}}\text{ and }R_2=\frac{-a\cos 2\theta}{\sqrt{\cos^2\theta+\sin^2\theta}}$$

$$\Rightarrow\ R_1^2=\frac{a^2}{\sec^2\theta+\text{cosec}^2\theta}=a^2\,\frac{1}{4}\sin^2 2\theta$$

$$\text{and }R_2^2=\frac{a^2\cos^2 2\theta}{\cos^2\theta+\sin^2\theta}=a^2\cos^2 2\theta$$

$$\therefore\ 4R_1^2+R_2^2=a^2(\sin^2 2\theta+\cos^2 2\theta)=a^2$$

7. **(b) :** The equation of the line is $3x-4y=56$ ...(i)

and that of the curve is

$x^2+y^2-24x+10y=0$ ...(ii)

Now, homogening the equation (i) and (ii), we get

$$x^2+y^2-(24x-10y)\left(\frac{3x-4y}{56}\right)=0$$

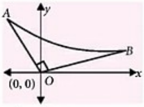
$\Rightarrow$ $56(x^2+y^2)-(72x^2-126xy+40y^2)=0$

$\Rightarrow$ $8x^2-63xy-8y^2=0$

$\Rightarrow$ $(8x+y)(x-8y)=0$

$\therefore$ $y=-8x$ and $x=8y$,

which are the equations of line OA and OB .



Clearly they are perpendicular to each other.

8. **(a) :** We observe that

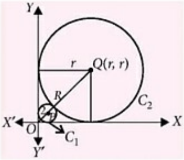
$OQ = OR + RQ$

$\Rightarrow \sqrt{2}r = OP + PR + RQ$

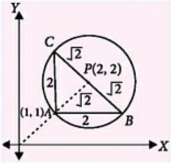
$\Rightarrow \sqrt{2}r = 2\sqrt{2} + 2 + r$

$\Rightarrow r = 2\left(\dfrac{\sqrt{2}+1}{\sqrt{2}-1}\right)$

$= 2(3+2\sqrt{2}) = 6 + 4\sqrt{2}$



9. **(b) :** Since the chord BC is perpendicular to AP and passes through the centre $P(2, 2)$, so its equation is $x + y = 4$.



10. **(c) :** The area of the circle $= \pi r^2 = 49\pi \Rightarrow r = 7$

Also the centre of the circle is the point of intersection of the diameters $3x - 4y - 7 = 0$ and $2x - 3y - 5 = 0$

Solving the above equation, we get centre $(1, -1)$.

$\therefore$ Equation of circle is $(x-1)^2 + (y+1)^2 = 49$

$\Rightarrow x^2 + y^2 - 2x + 2y - 47 = 0$

11. **(d) :** We have, ${}^{35}C_8 + \sum_{r=1}^{7} {}^{42-r}C_7 + \sum_{s=1}^{5} {}^{47-s}C_{40-s}$

$$= {}^{35}C_8 + {}^{35}C_7 + {}^{36}C_7 + {}^{37}C_7 + \ldots\ldots + {}^{41}C_7 + {}^{42}C_{35} + {}^{43}C_{36} + \ldots\ldots + {}^{46}C_{39}$$

$$= ({}^{35}C_8 + {}^{35}C_7) + {}^{36}C_7 + {}^{37}C_7 + \ldots\ldots + {}^{41}C_7 + {}^{42}C_{35} + {}^{43}C_{36} + \ldots\ldots + {}^{46}C_{39}$$

$$= ({}^{36}C_8 + {}^{36}C_7) + \ldots\ldots + {}^{41}C_7 + {}^{42}C_7 + {}^{43}C_7 + \ldots\ldots + {}^{46}C_7$$

$$[\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r \text{ and } {}^nC_r = {}^nC_{n-r}]$$

$$= ({}^{41}C_8 + {}^{41}C_7) + {}^{42}C_7 + {}^{43}C_7 + \ldots\ {}^{46}C_7$$

$$= ({}^{42}C_8 + {}^{42}C_7) + \ldots\ldots + {}^{46}C_7$$

$= \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$$= {}^{46}C_8 + {}^{46}C_7 = {}^{47}C_8$$

12. **(b) :** Given that 4 particular guests will sit on a particular side A (say) and three other on the other side B (say). Therefore, we are to arrange 11 guests so that 5 guests will sit on side A and remaining 6 will sit on side B. This can be done in ${}^{11}C_5 \times {}^6C_6$ ways.

Now, 9 guests on each side of table can arrange among themselves in $9! \times 9!$ ways.

Therefore, total number of arrangements

$$= {}^{11}C_5 \times {}^6C_6 \times 9! \times 9! = {}^{11}C_5 \times 9! \times 9!$$

13. **(b) :** The given equation of the circle is $x^2 + y^2 - x + 3y = 0$.

Centre and radius are $\left(\dfrac{1}{2}, -\dfrac{3}{2}\right)$ and $\sqrt{\dfrac{5}{2}}$ respectively.

Let $y = mx$ be the equation of L_1.

Then, p_1 = Length of the intercept on L_1.

$$\Rightarrow p_1 = 2\sqrt{\left(\sqrt{\frac{5}{2}}\right)^2 - \left(\frac{m+3}{2\sqrt{m^2+1}}\right)^2}$$

$$\Rightarrow p_1 = 2\sqrt{\frac{5}{2} - \frac{(m+3)^2}{4(m^2+1)}}$$

And p_2 = Length of the intercept on L_2.

$$\Rightarrow p_2 = 2\sqrt{\left(\sqrt{\frac{5}{2}}\right)^2 - (\sqrt{2})^2} \Rightarrow p_2 = 2\sqrt{\frac{5}{2} - 2} = \sqrt{2}$$

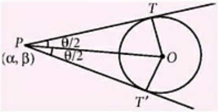
Now, $p_1 = p_2 \Rightarrow 2\sqrt{\dfrac{5}{2} - \dfrac{(m+3)^2}{4(m^2+1)}} = \sqrt{2}$

$$\Rightarrow 7m^2 - 6m - 1 = 0 \quad \Rightarrow \quad m = 1, -\frac{1}{7}$$

So, the equations of L_1 are $y = x$ and $7y = -x$.

14. **(b) :** Let PT and PT' be the tangents drawn from $P(\alpha, \beta)$ to the circle $x^2 + y^2 = a^2$, and let $\angle TPT' = \theta$. If O is the centre of the circle, then $\angle TPO = \angle T'PO = \dfrac{\theta}{2}$.

$$\therefore \tan\frac{\theta}{2} = \frac{OT}{TP} = \frac{a}{\sqrt{s_1}} \Rightarrow \theta = 2\tan^{-1}\left(\frac{a}{\sqrt{s_1}}\right)$$



15. **(b) :** Let ABC be the triangle such that its sides $a = BC$ and $b = CA$ are the roots of the equation $x^2 - 2\sqrt{3}x + 2 = 0$

$\therefore \quad a + b = 2\sqrt{3}$ and $ab = 2$

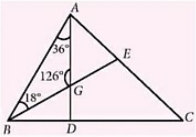
Also, $\angle C = \dfrac{\pi}{3} \Rightarrow \cos C = \dfrac{1}{2}$

$$\Rightarrow \frac{a^2 + b^2 - c^2}{2ab} = \frac{1}{2} \Rightarrow (a+b)^2 - c^2 = 3ab$$

$$\Rightarrow (2\sqrt{3})^2 - c^2 = 3 \times 2 \Rightarrow c^2 = 6 \Rightarrow c = \sqrt{6}$$

$\therefore \quad$ Perimeter of $\Delta ABC = a + b + c = 2\sqrt{3} + \sqrt{6}$

16. (b) : In the triangle ΔAGB, we have



$$\frac{AG}{\sin 18^\circ}=\frac{AB}{\sin 126^\circ}$$

$$\Rightarrow AB=\frac{\sin 126^\circ}{\sin 18^\circ}AG$$

$$\Rightarrow AB=\frac{\cos 36^\circ}{\sin 18^\circ}\times\frac{2}{3}AD$$

$$\Rightarrow AB=\frac{\sqrt{5}+1}{\sqrt{5}-1}\times\frac{2}{3}\times 6=2(3+\sqrt{5})$$

Now, $R=\dfrac{c}{2\sin C}\Rightarrow R=(3+\sqrt{5})\operatorname{cosec}C$

17. (c) : Let ABC be the isosceles triangle with $AB=AC$ and $\angle A=\dfrac{2\pi}{3}$.

Now, $\Delta=\dfrac{1}{2}(AB\times AC)\times\sin\dfrac{2\pi}{3}$

$\Rightarrow\ \Delta=\dfrac{\sqrt{3}}{4}x^2$, where $AB=AC=x$(say)

Now, $\dfrac{AB}{\sin 30^\circ}=\dfrac{BC}{\sin 120^\circ}=\dfrac{AC}{\sin 30^\circ}\Rightarrow 2x=\dfrac{2a}{\sqrt{3}}$

$\Rightarrow x=\dfrac{a}{\sqrt{3}}(a=BC)$

$\therefore\ \ 2s=2x+a\ \Rightarrow\ 2s=\dfrac{2a}{\sqrt{3}}+a$

$\Rightarrow\ s=\left(\dfrac{2+\sqrt{3}}{2\sqrt{3}}\right)a$ and $\Delta=\dfrac{\sqrt{3}}{4}\times\dfrac{a^2}{3}=\dfrac{a^2}{4\sqrt{3}}$

Now, $r=\dfrac{\Delta}{s}\Rightarrow\sqrt{3}=\dfrac{a^2}{4\sqrt{3}}\times\dfrac{2\sqrt{3}}{(2+\sqrt{3})a}$

$\Rightarrow a=6+4\sqrt{3}\ \therefore\ \Delta=12+7\sqrt{3}$

18. (b) : Here, the given inequality is

$\log_{\frac{1}{2}}|z-1|>\log_{\frac{1}{2}}|z-i|$

Here the base is $\dfrac{1}{2}\in(0,1)$

$\therefore\ \ |z-1|<|z-i|\Rightarrow|z-1|^2<|z-i|^2$

$\Rightarrow\ \ (z-1)(\bar{z}-1)<(z-i)(\bar{z}+i)\ [\because\ |z|^2=z\bar{z}]$

$\Rightarrow\ \ z\bar{z}-\bar{z}-z+1<z\bar{z}-i\bar{z}+zi-i^2$

$\Rightarrow\ \ i\bar{z}-\bar{z}-z-zi<0\ \Rightarrow\ z+\bar{z}+zi-\bar{z}i>0$

$\Rightarrow\ \ (z+\bar{z})+(z-\bar{z})i>0\ \Rightarrow\ \left(\dfrac{z+\bar{z}}{2}\right)+i\left(\dfrac{z-\bar{z}}{2}\right)>0$

$\Rightarrow\ \ \mathrm{Re}(z)-\mathrm{Im}(z)>0\Rightarrow a-b>0$ [since $z=a+ib$]

19. (c) : Given that $|z|=|\omega|=r$, let $\arg(\omega)=\theta$

$\therefore\ \ \omega=r(\cos\theta+i\sin\theta)$

and $\arg(z)+\arg(\omega)=\pi\Rightarrow\arg(z)=\pi-\theta$

$\therefore\ \ z=r(\cos(\pi-\theta)+i\sin(\pi-\theta))$

$\quad=r(-\cos\theta+i\sin\theta)=-r(\cos\theta-i\sin\theta)=-\bar{\omega}$

20. (c) : Let the roots of the given equation be α,β.

$\Rightarrow\ \ \alpha+\beta=\omega$ and $\alpha\beta=\nu$

Also given that $\alpha-\beta=2$

Now $(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta$

$\Rightarrow\ \ 4=\omega^2-4\nu\Rightarrow\omega^2=4(1+\nu)$

21. (20) : The area of the triangle formed by z, iz, $z+iz$ is given by $\dfrac{1}{2}|z|^2$.

$\therefore\ \ \dfrac{1}{2}|z|^2=200\ \Rightarrow\ |z|^2=400\ \Rightarrow\ |z|=20$

……

SAMURAI SUDOKU

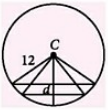
Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

The puzzle has a unique answer.

		3		4				7						2			4		9	
						2						5			8				1	
9			1		6	5							7				3			6
	3		7			6							8	1	2					7
6							1	8					6	7		4	8	2	3	
	4			7			2						4	9	7					2
			5	8		1			3	4				8			9			
	7	6		2			5		6								2	5		9
							6				8			2						
								5				3		1						
						3			2					7						
		7	6				1				3		7				1			6
					3		8	4								6			7	2
2	6	8		5					5		4				9					
				3				9				4			6					7
4	8			2									5						8	
		3				1	7						6	8		2	4		9	
					2	7	9	6					9	1	3				4	
6	7				9	4						2							6	5
				1												4		1		

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

22. (6) : Distance, d



$= 12(\cos 36° - \cos 72°)$
$= 24\sin 54° \sin 18°$
$= 24\cos 36° \sin 18°$

$$= 24 \cdot \frac{\sqrt{5}+1}{4} \cdot \frac{\sqrt{5}-1}{4} = 6 \text{ units}$$

23. (0) : Here $f(x) = x^{6a} + x^{6b+1} + x^{6c+2}$

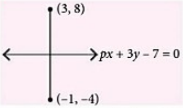
$\therefore\ f(\omega) = (\omega^6)^a + \omega(\omega^6)^b + \omega^2(\omega^6)^c$

$\Rightarrow f(\omega) = 1 + \omega + \omega^2 = 0$

24. (16) : The number of triangles = Total number of triangles – (number of triangles having one side common with the octagon) – (number of triangles having two sides common with the octagon)

$= {}^8C_3 - ({}^8C_1 \times {}^4C_1) - 8 = 16$

25. (1) :



Since, image of point (x, y) from line $ax + by + c = 0$ is given by

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{-2(ax_1+by_1+c)}{a^2+b^2}$$

$$\therefore\ \frac{-1-3}{p} = \frac{-4-8}{3} = \frac{-2(3p+24-7)}{p^2+9}$$

$$\therefore\ \frac{-4}{p} = \frac{-12}{3} \Rightarrow p = 1$$

26. (614) : In a dictionary the words at each stage are arranged in alphabetical order. So, here we consider the words beginning with A, D, M, N, O and R in order.

Now keeping A at the first place, the remaining 5 letters can be arranged in 5! ways. Similarly, D, M, N, O will occur in the first place the same number of times *i.e.*, 5!

$\therefore$ Number of words starting with A = 5! = 120
Number of words starting with D = 5! = 120
Number of words starting with M = 5! = 120
Number of words starting with N = 5! = 120
Number of words starting with O = 5! = 120

Number of words beginning with R is 5!, but one of these words is the word RANDOM. So, we first find the number of words beginning with RAD and RAM.

Number of words starting with RAD = 3! = 6
and number of words starting with RAM = 3! = 6.

Now, the words beginning with RAN must follow. The first word beginning with RAN is the word RANDMO and the next word is RANDOM.

$\therefore$ Rank of RANDOM = 5 × 120 + 2 × 6 + 2 = 614

27. (45) : Let $b_1, b_2, \ldots\ldots, b_5$ be five balls and $B_1, B_2, \ldots\ldots, B_5$ be five boxes. Then, the required number of ways

$$= \left[\sum_{r=1}^{5} (r^{\text{th}} \text{ ball is placed in } r^{\text{th}} \text{box})\right]$$
$$\times \text{ (Remaining balls are placed in wrong boxes)}$$

$$= \sum_{r=1}^{5} 1 \times 4! \left\{1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right\}$$

$$= \sum_{r=1}^{5} (12 - 4 + 1) = 45$$

28. (1) : The nearest point on the lines $3x + 4y = 25$ ...(i) from the origin is the foot of the perpendicular from the origin to the line (i).

Now, let the perpendicular to equation (i) is

$4x - 3y + k = 0$ ...(ii)

Since (ii) is passing through (0,0) $\therefore k = 0$

So (ii) becomes $4x - 3y = 0$ ...(iii)

Now solving (i) and (iii), we get the required point, which is (3, 4) *i.e.*, (a, b)

$\therefore\ b - a = 4 - 3 = 1$

29. (57) : The equation of the given circle is

$x^2 + y^2 - 6x - 10y + \lambda = 0$

Given that the circle does not cut the coordinate axes

$\therefore\ r \text{ (radius)} < 3 \Rightarrow (3)^2 + (5)^2 - \lambda < 9 \Rightarrow \lambda > 25$ ...(i)

Again, the point (2, 4) lies inside the circle.

$\therefore\ 4 + 16 - 12 - 40 + \lambda < 0 \Rightarrow \lambda < 32$ ...(ii)

From (i) and (ii), we get $\lambda \in (25, 32)$ *i.e.*, (a, b)

$\therefore\ a + b = 25 + 32 = 57$

30. (399960) : We know that the sum of all n-digit numbers formed by using n digits from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 = (Sum of the digits) $(n-1)!\left(\frac{10^n - 1}{10 - 1}\right)$

$$\therefore\ \text{Required sum} = 4!\,(1+2+3+4+5) \times \left(\frac{10^5 - 1}{10 - 1}\right)$$

$$= 360\left(\frac{100000 - 1}{9}\right) = 3999960$$

◆◆

Monthly Test Drive CLASS XI ANSWER KEY

1. (b)	**2.** (b)	**3.** (c)	**4.** (b)	**5.** (c)
6. (a)	**7.** (a,b)	**8.** (a,c)	**9.** (b,c)	**10.** (c)
11. (a,c)	**12.** (c)	**13.** (a,b)	**14.** (a)	**15.** (a)
16. (b)	**17.** (12)	**18.** (7)	**19.** (7)	**20.** (2)

Exam Dates 2021
27th to 30th April
24th to 28th May

FOCUS JEE MAIN 2021

PRACTICE QUESTIONS

MULTIPLE CHOICE QUESTIONS

1. If $x_1, x_2, ..., x_n$ are in H.P., then $\sum_{r=1}^{n-1} x_r x_{r+1}$ is equal to

(a) $(n-1)x_1x_n$ (b) nx_1x_n
(c) $(n+1)x_1x_n$ (d) none of these

2. The value of $\sum_{r=0}^{n} (r)\,{}^{2n}C_r$ is equal to

(a) $2n(2^{2n-2} + {}^{2n-1}C_{n-1})$
(b) $n(2^{2n-2} + {}^{2n-1}C_n)$
(c) $2n(2^{2n-2} + {}^{2n-1}C_{n+1})$
(d) none of these

3. Total number of ways, in which 22 different books can be given to 5 students, so that two students get 5 books each and all the remaining students get 4 books each, is equal to

(a) $\dfrac{22!}{3!2!5!(4!)^3}$ (b) $\dfrac{22!}{(3!)^2 \cdot 2! \cdot 5!}$
(c) $\dfrac{22!}{3!\,2!\,5!4!}$ (d) none of these

4. Let $\vec{a}, \vec{b}, \vec{c}$ be three unit vectors such that $\vec{a} + 5\vec{b} + 3\vec{c} = \vec{0}$, then $\vec{a} \cdot (\vec{b} \times \vec{c})$ is equal to

(a) $\vec{a} \cdot \vec{b}$ (b) $\vec{a} \cdot (\vec{b} + 2\vec{c})$
(c) $\vec{b} \cdot (\vec{a} + \vec{c})$ (d) none of these

5. Consider the set of integers {10, 11, 12, ..., 98, 99}. By seeing the number a person A will laugh if the product of the digits of the number is 12. He chooses three numbers one at a time from this set of integers, randomly and with replacement. The probability that he will laugh atleast once, is equal to

(a) $1-\left(\dfrac{43}{45}\right)^3$ (b) $1-\left(\dfrac{3}{5}\right)^3$
(c) $1-\left(\dfrac{4}{25}\right)^3$ (d) none of these

6. If A and B are square matrices such that $AB = B$ and $BA = A$, then $A^2 + B^2$ is always equal to

(a) $2AB$ (b) $2BA$ (c) $A + B$ (d) AB

7. Range of $f(x) = \cos^{-1}\left(\dfrac{1+x^2}{2x}\right) + \sqrt{2-x^2}$ is

(a) $\left\{0, 1+\dfrac{\pi}{2}\right\}$ (b) $\{0, 1+\pi\}$
(c) $\left\{1, 1+\dfrac{\pi}{2}\right\}$ (d) $\{1, 1+\pi\}$

8. If $[\cos^{-1}x] + [\cot^{-1}x] = 0$, where [.] denotes the greatest integer function, then complete set of values of x is

(a) $(\cos 1, 1]$ (b) $(\cos 1, \cot 1)$
(c) $(\cot 1, 1]$ (d) $[0, \cot 1)$

9. $f(x) = x^2 - 4|x|$ and

$$g(x) = \begin{cases} \min\{f(t) : -6 \le t \le x\}, x \in [-6, 0] \\ \max\{f(t) : 0 < t \le x\}, x \in (0, 6] \end{cases}$$ then $g(x)$ has

(a) exactly one point of local minima
(b) exactly one point of local maxima
(c) no point of local maxima but exactly one point of local minima
(d) neither a point of local maxima nor minima

10. $\int_0^{\frac{3\pi}{2}} \sin\left[\frac{2x}{\pi}\right]dx$, where $[\cdot]$ denotes the greatest integer function, is equal to

(a) $\frac{\pi}{2}(\sin 1 + \cos 1)$ (b) $\frac{\pi}{2}(\sin 1 - \sin 2)$

(c) $\frac{\pi}{2}(\sin 1 - \cos 1)$ (d) $\frac{\pi}{2}(\sin 1 + \sin 2)$

11. General solution of $x\frac{dy}{dx} + y = y^2x^3\cos x$, is

(a) $\frac{1}{xy} + x\sin x - \cos x = C$

(b) $\frac{1}{xy} - x\sin x + \cos x = C$

(c) $\frac{1}{xy} + x\cos x - \sin x = C$

(d) $\frac{1}{xy} + x\sin x + \cos x = C$

12. A variable line is drawn through the intersection point of the lines $3x + 4y - 12 = 0$ and $x + 2y - 5 = 0$, meeting the co-ordinate axes at the points A and B. Locus of midpoint of segment AB is

(a) $4x + 3y = 4xy$ (b) $3x + 4y = 3xy$

(c) $3x + 4y = 4xy$ (d) none of these

13. If $x - 2y + 4 = 0$ and $2x + y - 5 = 0$ are the sides of an isosceles triangle having area 10 sq. units, then equation of third side is

(a) $3x - y = -9$ (b) $3x - y + 11 = 0$

(c) $x - 3y = 10$ (d) $3x - y + 15 = 0$

14. PA and PB are tangents drawn to $y^2 = 4ax$ from arbitrary point P. If the angle between tangents is $\pi/4$, then locus of point P is

(a) $y^2 = x^2 + a^2 + 6ax$ (b) $y^2 = x^2 - a^2 + 6ax$

(c) $y^2 = x^2 + a^2 - 6ax$ (d) $y^2 = x^2 - a^2 - 6ax$

15. The equation of common tangents of the curves $x^2 + 4y^2 = 8$ and $y^2 = 4x$ are

(a) $y - 2x - 4 = 0, y + 2x + 4 = 0$

(b) $y - 2x - 2 = 0, y + 2x + 2 = 0$

(c) $2y - x - 4 = 0, 2y + x + 4 = 0$

(d) $2y - x - 2 = 0, 2y + x + 2 = 0$

16. The equation of the plane passing through the points $(2, -1, 0)$, $(3, -4, 5)$ and parallel to the line $2x = 3y = 4z$ is

(a) $125x - 90y - 79z = 340$

(b) $32x - 21y - 36z = 85$

(c) $73x + 61y - 22z = 85$

(d) $29x - 27y - 22z = 85$

17. Reflection of line $\frac{x-1}{-1} = \frac{y-2}{3} = \frac{z-4}{1}$ in the plane $x + y + z = 7$ is

(a) $\frac{x-1}{3} = \frac{y-2}{1} = \frac{z-4}{1}$

(b) $\frac{x-1}{-3} = \frac{y-2}{-1} = \frac{z-4}{1}$

(c) $\frac{x-1}{-3} = \frac{y-2}{1} = \frac{z-4}{-1}$

(d) $\frac{x-1}{3} = \frac{y-2}{1} = \frac{z-4}{-1}$

18. If x, y, z are in A.P., then $\frac{\sin x - \sin z}{\cos z - \cos x}$ is equal to

(a) $\tan y$ (b) $\cot y$ (c) $\sin y$ (d) $\cos y$

19. $\sum_{r=1}^{n} \sin^{-1}\left(\frac{\sqrt{r} - \sqrt{r-1}}{\sqrt{r(r+1)}}\right)$ is equal to

(a) $\tan^{-1}(\sqrt{n}) - \frac{\pi}{4}$ (b) $\tan^{-1}(\sqrt{n+1}) - \frac{\pi}{4}$

(c) $\tan^{-1}(\sqrt{n})$ (d) none of these

20. Equation of the plane such that foot of altitude drawn from $(-1, 1, 1)$ to the plane has the coordinate $(3, -2, -1)$, is

(a) $x + y + z = 0$ (b) $4x - 3y - 2z = 20$

(c) $3x + y - z = 8$ (d) $4x + 3y - z = 7$

21. If a, b, c are the sides of a triangle ABC, then $\sqrt{a} + \sqrt{b} - \sqrt{c}$ is always

(a) negative (b) positive

(c) non-negative (d) non-positive

22. $\int \tan^{-1}\sqrt{x}\,dx$ is equal to

(a) $(x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + C$

(b) $x\tan^{-1}\sqrt{x} - \sqrt{x} + C$

(c) $\sqrt{x} - x\tan^{-1}\sqrt{x} + C$

(d) $\sqrt{x} - (x+1)\tan^{-1}\sqrt{x} + C$

23. If $f(x) = 2e^x - ae^{-x} + (2a + 1)x - 3$ is increasing for all values of x, then

(a) $a \in (-\infty, \infty)$ (b) $a \in (-\infty, 0]$

(c) $a \in [0, \infty)$ (d) none of these

24. Domain of $f(x)=\sqrt{[x]-1+x^2}$; where $[\cdot]$ denotes the greatest integer function, is

(a) $(-\infty, -2) \cup [1, \infty)$ (b) $(-\infty, -\sqrt{2}) \cup [1, \infty)$

(c) $(-\infty, -3) \cup [1, \infty)$ (d) $(-\infty, -\sqrt{3}] \cup [1, \infty)$

25. $\sum_{r=1}^{n} \frac{r}{(r+1)!}$ is equal to

(a) $1-\frac{1}{(n+1)!}$ (b) $1-\frac{1}{n!}$

(c) $1-\frac{1}{(n-1)!}$ (d) none of these

NUMERICAL VALUE TYPE

26. Total number of real values of x, such that $\frac{(12+x)^{1/7}}{x}+\frac{(12+x)^{1/7}}{12}=\frac{64}{3}x^{1/7}$ is _______.

27. If the equations $x^2 + ax + 12 = 0$, $x^2 + bx + 15 = 0$ and $x^2 + (a + b)x + 36 = 0$ have a common positive root, then ab equals _______.

28. If $|z| \le 4$, then the maximum value of $|iz + 3 - 4i|$ is equal to _______.

29. A team of four students is to be selected from a total of 12 students. Total number of ways in which team can be selected such that two particular students refuse to be together and other two particular students wish to be together only, is equal to _______.

30. In triangle ABC, the absolute value of

$$\begin{vmatrix} e^{-i2A} & e^{iC} & e^{iB} \\ e^{iC} & e^{-i2B} & e^{iA} \\ e^{iB} & e^{iA} & e^{-i2C} \end{vmatrix}$$

, where $i=\sqrt{-1}$, is equal to ____.

31. The probability that an archer hits the target when it is windy is equal to 2/5. When it is not windy his probability of hitting the target is 7/10. On any shot the probability of gust of wind is 3/10. The probability that there is no gust of wind on the occasion when he missed the target, is equal to $7/\lambda$, where λ equals _______.

32. $\lim_{x\to 1} \frac{(x-1)(x^2-1)(x^3-1)(x^4-1)}{((x-1)(x^2-1))^2}$ is equal to ____.

33. $f(x) = [\sin x] + [\cos x]$, $x \in [0, 2\pi]$, where $[\cdot]$ denotes the greatest integer function. Total number of points where $f(x)$ is non-differentiable is equal to ____.

34. If $\int \sin x \cdot \cos x \cdot \cos 2x \cdot \cos 4x \cdot \cos 8x \, dx$ is equal to $-\frac{1}{\lambda}\cos 16x + c$, then λ equals _______.

35. Value of positive real parameter 'a' such that the area of the region bounded by the parabola $y = x - ax^2$, $ay = x^2$ attains its maximum value, is equal to _______.

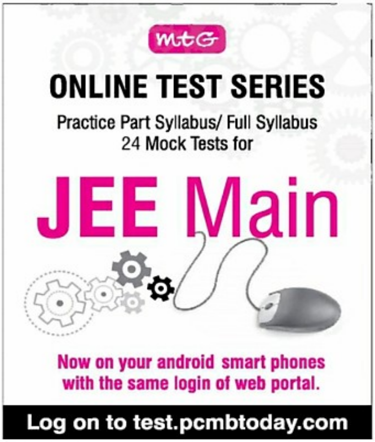
36. The circle $x^2 + y^2 - 4x - 4y + 4 = 0$ is inscribed in a variable triangle OAB. Sides OA and OB lie along the x and y-axis respectively, where 'O' is the origin. Locus of midpoint of side AB is $x+y-xy+\lambda\sqrt{x^2+y^2}=0$, then λ is equal to _______.

37. If the length of the latus rectum of the hyperbola $xy = c^2$, is equal to $2\sqrt{pc}$, then p equals _______.

38. If the minimum value of $27^{\cos x} + 81^{\sin x}$ is equal to $\frac{\lambda}{9\sqrt{3}}$, then λ equals _______.

39. If in triangle ABC, $\angle A = \pi/2$ then the value of $r + 2R$ is equal to $\frac{1}{p}(a + b + c)$, where p is _______.

40. If the curves $\frac{x^2}{a^2}+\frac{y^2}{4}=1$ and $y^2 = 16x$ intersect at right angle, then a^2 is equal to _______.



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SOLUTIONS

1. (a) : Clearly, $\frac{1}{x_1}, \frac{1}{x_2}, ..., \frac{1}{x_n}$ will be in A.P.

$$\Rightarrow \frac{1}{x_2}-\frac{1}{x_1}=\frac{1}{x_3}-\frac{1}{x_2}=.....=\frac{1}{x_{r+1}}-\frac{1}{x_r}=.....$$

$$=\frac{1}{x_n}-\frac{1}{x_{n-1}}=\lambda \text{ (say)}$$

$$\Rightarrow \sum_{r=1}^{n-1} x_r x_{r+1}=\frac{1}{\lambda}\sum_{r=1}^{n-1}(x_r-x_{r+1})=\frac{1}{\lambda}(x_1-x_n)$$

Now, $\frac{1}{x_n}=\frac{1}{x_1}+(n-1)\lambda \Rightarrow \frac{x_1-x_n}{x_1x_n}=(n-1)\lambda$

$$\Rightarrow \sum_{r=1}^{n-1} x_r x_{r+1}=(n-1)x_1x_n$$

2. (d) : We have, $\sum_{r=0}^{n} r\cdot {}^{2n}C_r = \sum_{r=0}^{n} 2n\cdot {}^{2n-1}C_{r-1}$

$= 2n({}^{2n-1}C_0 + {}^{2n-1}C_1 + ... + {}^{2n-1}C_{n-1})$

$= 2n(\frac{1}{2}(2^{2n-1})) = 2n(2^{2n-2})$

3. (a) : It is same as dividing 22 books in 5 packets such that two packets have 5 books each and remaining three packets have four books each and then distributing these packets.

Thus, total ways of distributing these books

$$=\frac{22!}{(5!)^2\cdot(4!)^3\cdot 2!3!}.(5!)=\frac{22!}{2!3!5!(4!)^3}$$

4. (d) : $\vec{a}+5\vec{b}+3\vec{c}=\vec{0}$

Thus $\vec{a}, \vec{b}, \vec{c}$ are coplanar. Hence $\vec{a}\cdot(\vec{b}\times\vec{c})=0$

5. (a) : Person will laugh if the selected number is 43 or 34 or 62 or 26.

Thus, probability that person will not laugh on seeing the number $=\frac{86}{90}=\frac{43}{45}$

Thus, required probability $= 1-\left(\frac{43}{45}\right)^3$

6. (c) : Given, $AB = B$, $BA = A$

Now, $A^2 + B^2 = AA + BB$

$= A(BA) + B(AB) = (AB)A + (BA)B$

$= BA + AB = A + B$

7. (d) : $\cos^{-1}\left(\frac{1+x^2}{2x}\right)$ is defined if $\left|\frac{1+x^2}{2x}\right|\le 1$

$\Rightarrow x \ne 0 \Rightarrow 1 + x^2 - 2|x| \le 0$

$\Rightarrow (|x| - 1)^2 \le 0 \Rightarrow x = 1, -1$

Thus, domain of $f(x)$ is $\{1, -1\}$.

Hence, range is $\{1, 1 + \pi\}$

8. (c) : We have, $[\cos^{-1}x] \ge 0\ \forall\ x \in [-1, 1]$

and $[\cot^{-1}x] > 0\ \forall\ x \in R$

Now, $[\cos^{-1}x] + [\cot^{-1}x] = 0$

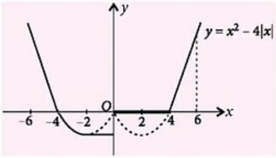
$\Rightarrow [\cos^{-1}x] = [\cot^{-1}x] = 0$

If $[\cos^{-1}x] = 0$, then $x \in (\cos 1, 1]$

If $[\cot^{-1}x] = 0$, then $x \in (\cot 1, \infty)$

$\Rightarrow x \in (\cot 1, 1]$

9. (d) : Bold line represents the graph of $y = g(x)$. Clearly, $g(x)$ has neither a point of local maxima nor a point of local minima.



10. (d) : $\int_0^{3\pi/2} \sin\left[\frac{2x}{\pi}\right]dx$

$$=\int_0^{\pi/2} \sin\left[\frac{2x}{\pi}\right]dx+\int_{\pi/2}^{\pi} \sin\left[\frac{2x}{\pi}\right]dx+\int_{\pi}^{3\pi/2} \sin\left[\frac{2x}{\pi}\right]dx$$

$$=0+\sin 1\int_{\pi/2}^{\pi} dx+\sin 2\int_{\pi}^{3\pi/2} dx \ =\frac{\pi}{2}(\sin 1+\sin 2)$$

11. (d) : Given, $x.\frac{dy}{dx}+y=y^2\cdot x^3\cdot\cos x$

$$\Rightarrow \frac{(xdy+ydx)}{x^2y^2}=x\cdot\cos x dx$$

$$\Rightarrow \frac{d(xy)}{x^2y^2}=x\cdot\cos x dx \Rightarrow -\frac{1}{xy}=x\sin x+\cos x+C$$

12. (c) : Equation of any line drawn through the intersection point of the given lines is,

$$3x + 4y - 12 + \lambda(x + 2y - 5) = 0$$

$$\Rightarrow A\equiv\left(\frac{5\lambda+12}{3+\lambda}, 0\right), B\equiv\left(0, \frac{5\lambda+12}{2\lambda+4}\right)$$

If midpoint of AB is $C(h, k)$, then

$$2h=\frac{5\lambda+12}{3+\lambda},\ 2k=\frac{5\lambda+12}{2\lambda+4}$$

$$\Rightarrow \frac{h}{k}=\frac{2(\lambda+2)}{3+\lambda} \Rightarrow \lambda=\frac{4k-3h}{(h-2k)}$$

Thus, $2h=\dfrac{12+\dfrac{5(4k-3h)}{(h-2k)}}{3+\dfrac{4k-3h}{h-2k}}$

Thus, required locus is, $3x + 4y = 4xy$.

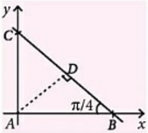
13. (a) : Given lines are mutually perpendicular and intersect at $\left(\frac{6}{5},\frac{13}{5}\right)$.

Equation of angle bisector of the given lines are

$$x - 2y + 4 = \pm(2x + y - 5)$$

i.e., $x + 3y = 9$, $3x - y = 1$

Side BC will be parallel to one of these bisectors.



Let $AD = a \Rightarrow AB = a\sqrt{2}$

and, area of $\Delta_{ABC}=\frac{1}{2}(a\sqrt{2})^2$

$\Rightarrow a^2 = 10 \quad \Rightarrow a=\sqrt{10}$

Let equation of BC is $x + 3y = \lambda$

$$\Rightarrow \sqrt{10}=\frac{\left|\frac{6}{5}+\frac{39}{5}-\lambda\right|}{\sqrt{10}} \Rightarrow \lambda = -1, 19$$

Thus equation of BC is

$$x + 3y = -1 \text{ or } x + 3y = 19$$

If equation of BC is $3x - y = \lambda$

$$\Rightarrow \sqrt{10}=\frac{\left|\frac{18}{5}-\frac{13}{5}-\lambda\right|}{\sqrt{10}} \Rightarrow \lambda = -9, 11$$

Hence, equation of BC is

$$3x - y = -9 \text{ or } 3x - y = 11$$

14. (a) : Let $P \equiv (h, k)$

Tangent in terms of m is $y=mx+\frac{a}{m}$

$\Rightarrow m^2x - my + a = 0$

If it passes through (h, k), then

$$m^2h - mk + a = 0$$

Since, angle between these tangents is $\pi/4$.

$$\therefore \tan\frac{\pi}{4}=\frac{|m_1-m_2|}{|1+m_1m_2|}$$

$\Rightarrow (m_1 - m_2)^2 = (1 + m_1m_2)^2$

$\Rightarrow (m_1 + m_2)^2 - 4m_1m_2 = (1 + m_1m_2)^2$

$$\Rightarrow \frac{k^2}{h^2}-\frac{4a}{h}=\left(1+\frac{a}{h}\right)^2$$

Thus, required locus is, $y^2 = 4ax + (a + x)^2$.

15. (c) : Tangent in terms of 'm' for $y^2 = 4x$ is $y=mx+\frac{1}{m}$

and for $\frac{x^2}{8}+\frac{y^2}{2}=1$ tangent is, $y=mx\pm\sqrt{8m^2+2}$

For common tangents, we must have $\frac{1}{m}=\pm\sqrt{8m^2+2}$

$\Rightarrow m=\pm\frac{1}{2}$

Thus, common tangents are $y=\frac{x}{2}+2$ and $y=-\frac{x}{2}-2$

i.e. $2y - x - 4 = 0$ and $2y + x + 4 = 0$

16. (d) : Given line is $2x = 3y = 4z$

i.e., $\frac{x}{6}=\frac{y}{4}=\frac{z}{3}$

Let the plane be $ax + by + cz = 1$

$\therefore \; 6a + 4b + 3c = 0$, $2a - b = 1$ and $3a - 4b + 5c = 1$

$$\Rightarrow a=\frac{29}{85},\ b=-\frac{27}{85},\ c=-\frac{22}{85}$$

Thus, equation of plane is $29x - 27y - 22z = 85$.

17. (c) : Given line passes through $P(1, 2, 4)$ and this point also lies on the given plane.

Thus, required line will be of the form

SAMURAI SUDOKU

ANSWER · MARCH 2021

7	3	5	8	1	9	6	4	2				3	4	7	9	5	8	2	6	1
9	8	6	2	3	4	7	1	5				5	1	6	2	7	4	9	3	8
4	1	2	7	6	5	9	3	8				2	8	9	1	3	6	5	7	4
2	7	9	5	8	1	4	6	3				1	3	4	5	9	2	7	8	6
3	5	8	9	4	6	2	7	1				7	6	2	8	4	3	1	9	5
6	4	1	3	7	2	5	8	9				8	9	5	7	6	1	4	2	3
5	6	4	1	9	8	3	2	7	6	4	8	9	5	1	3	8	7	6	4	2
1	9	3	6	2	7	8	5	4	1	2	9	6	7	3	4	2	5	8	1	9
8	2	7	4	5	3	1	9	6	7	5	3	4	2	8	6	1	9	3	5	7
						5	6	9	3	7	4	8	1	2						
						2	8	1	9	6	5	3	4	7						
						7	4	3	8	1	2	5	6	9						
4	8	7	5	1	9	6	3	2	5	8	7	1	9	4	7	3	6	5	2	8
9	5	3	6	2	7	4	1	8	2	9	6	7	3	5	4	8	2	1	6	9
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3	7	8	1	9	4	5	2	6				5	4	3	6	2	9	8	1	7
6	4	2	8	7	5	1	9	3				6	7	9	5	1	8	3	4	2
5	9	1	3	6	2	8	4	7				8	2	1	3	7	4	9	5	6
8	6	4	2	3	1	7	5	9				3	1	8	2	4	7	6	9	5
7	3	5	9	4	8	2	6	1				4	6	7	8	9	5	2	3	1
1	2	9	7	5	6	3	8	4				9	5	2	1	6	3	7	8	4

$$\frac{x-1}{l}=\frac{y-2}{m}=\frac{z-4}{n}$$

Any point on the given line is

$(-r_1+1, 3r_1+2, r_1+4)$.

If $r_1 = 1$, this point becomes $P \equiv (0, 5, 5)$.

Let $Q(a, b, c)$ be the reflection of 'P' in the given plane,

then $\frac{a}{2}+\frac{b+5}{2}+\frac{c+5}{2}=7$

i.e., $a+b+c=4$ and $\frac{a}{1}=\frac{b-5}{1}=\frac{c-5}{1}=\lambda$ (say)

$\Rightarrow a=\lambda, b=5+\lambda, c=5+\lambda$

$\Rightarrow 10+3\lambda=4 \Rightarrow \lambda=-2.$

Thus, $Q \equiv (-2, 3, 3)$

Hence, direction ratios of reflected line are $-3, 1, -1$.

Thus, its equation is $\frac{x-1}{-3}=\frac{y-2}{1}=\frac{z-4}{-1}$.

18. (b) : $\frac{\sin x-\sin z}{\cos z-\cos x}=\frac{2\cos\left(\frac{x+z}{2}\right).\sin\left(\frac{x-z}{2}\right)}{2\sin\left(\frac{x+z}{2}\right).\sin\left(\frac{x-z}{2}\right)}$

$=\cot\left(\frac{x+z}{2}\right)=\cot(y)$

19. (c) : $\sin^{-1}\left(\frac{\sqrt{r}-\sqrt{r-1}}{\sqrt{r(r+1)}}\right)=\tan^{-1}\left(\frac{\sqrt{r}-\sqrt{r-1}}{1+\sqrt{r(r-1)}}\right)$

$=\tan^{-1}\sqrt{r}-\tan^{-1}(\sqrt{r-1})$

$\Rightarrow \sum_{r=1}^{n}\sin^{-1}\left(\frac{\sqrt{r}-\sqrt{r-1}}{\sqrt{r(r+1)}}\right)=\sum_{r=1}^{n}\left(\tan^{-1}\sqrt{r}-\tan^{-1}\sqrt{r-1}\right)$

$=\tan^{-1}\sqrt{n}$

20. (b) : Clearly, the direction ratios of normal to the plane are $4, -3, -2$

Thus equation of plane will be $4x-3y-2z=d$

It will necessarily pass through $(3, -2, -1)$

i.e., $d=12+6+2=20$

Thus the equation of plane is $4x-3y-2z=20$.

21. (b) : Clearly, $a+b>c$

$\Rightarrow (\sqrt{a}+\sqrt{b})^2-2\sqrt{ab}>(\sqrt{c})^2$

$\Rightarrow (\sqrt{a}+\sqrt{b})^2>(\sqrt{c})^2 \Rightarrow \sqrt{a}+\sqrt{b}>\sqrt{c}$

$\Rightarrow \sqrt{a}+\sqrt{b}-\sqrt{c}>0$

22. (a) : Let $I=\int\tan^{-1}\sqrt{x}\,dx$

$=x\cdot\tan^{-1}\sqrt{x}-\int\frac{1}{1+x}.\frac{1}{2\sqrt{x}}.x\,dx$

$=x\cdot\tan^{-1}\sqrt{x}-\frac{1}{2}\int\frac{\sqrt{x}}{1+x}dx$

Now, putting $\sqrt{x}=t \Rightarrow \frac{1}{2\sqrt{x}}dx=dt$

i.e., $dx=2t\,dt$, we get

$I=x\cdot\tan^{-1}\sqrt{x}-\frac{1}{2}\int\frac{t}{1+t^2}2t\,dt$

$=x\tan^{-1}\sqrt{x}-\int\frac{t^2+1-1}{1+t^2}dt$

$=x\tan^{-1}\sqrt{x}-t+\tan^{-1}t+C$

$=x\tan^{-1}\sqrt{x}-\sqrt{x}+\tan^{-1}\sqrt{x}+C$

23. (c) : We have, $f'(x)=2e^x+ae^{-x}+2a+1$

$=e^{-x}(2e^{2x}+(2a+1)e^x+a)$

$=2e^{-x}\left(e^{2x}+\left(a+\frac{1}{2}\right)e^x+\frac{a}{2}\right)$

$=2e^{-x}(e^x+a)\left(e^x+\frac{1}{2}\right)$

For $f(x)$ to be increasing, $f'(x)\geq 0\ \forall\ x\in R$

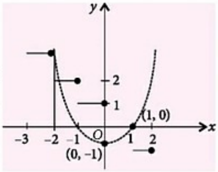
$\Rightarrow e^x+a\geq 0\ \forall\ x\in R \Rightarrow a\geq 0$

24. (d) : We have, $[x]-1+x^2\geq 0$

$\Rightarrow x^2-1\geq -[x]$

The graphs of $y=x^2-1$ and $y=-[x]$ intersect somewhere in $(-2, 1)$. Thus at the point of intersection, $x^2-1=2$

$\Rightarrow x=-\sqrt{3}$



From the adjacent graph, we get domain as

$(-\infty, -\sqrt{3}]\cup[1, \infty)$.

25. (a) : $\frac{r}{(r+1)!}=\frac{r+1-1}{(r+1)!}=\frac{1}{r!}-\frac{1}{(r+1)!}$

$=-\left(\frac{1}{(r+1)!}-\frac{1}{r!}\right)=-(V(r)-V(r-1))$

$\sum_{r=1}^{n}\frac{r}{(r+1)!}=-(V(n)-V(0))=1-\frac{1}{(n+1)!}$

26. (1) : $\frac{(12+x)^{1/7}}{x}+\frac{(12+x)^{1/7}}{12}=\frac{64}{3}x^{1/7}$

$\Rightarrow (12+x)(12+x)^{1/7}=64\cdot 4\cdot x\cdot x^{1/7}=2^8\cdot x^{8/7}$

$\Rightarrow 12+x=x(2^8)^{7/8} \Rightarrow x=\frac{12}{127}$

27. (56) : We have, $x^2+ax+12=0$ ...(i)

$x^2+bx+15=0$ ...(ii)

$x^2+(a+b)x+36=0$ ...(iii)

Adding (i) & (ii), we get

$$2x^2 + (a + b)x + 27 = 0 \qquad \text{...(iv)}$$

Subtracting (iii) from (iv), we get

$$x^2 - 9 = 0 \Rightarrow x = 3, -3.$$

Thus, common positive root is 3.

$\therefore\ 9 + 12 + 3a = 0 \Rightarrow a = -7$

and $9 + 3b + 15 = 0 \Rightarrow b = -8$

$\therefore\ ab = 56$

28. (9) : $|iz + 3 - 4i| \le |iz| + |3 - 4i| = |z| + 5 \le 9$

29. (226) : Let S_1 and S_2 refuse to be together and S_3 and S_4 want to be together only.

Total ways when S_3 and S_4 are selected

$= ({}^8C_2 + {}^2C_1 \cdot {}^8C_1) = 44$

Total ways when S_3 and S_4 are not selected

$= ({}^8C_4 + {}^2C_1 \cdot {}^8C_3) = 182$

Thus, total ways = 44 + 182 = 226

30. (b) : Given determinant

$$= e^{-i2A}(e^{-i2(B+C)} - e^{i2A}) - e^{iC}(e^{-iC} - e^{i(A+B)}) + e^{iB}(e^{i(A+C)} - e^{-iB})$$

$$= e^{-i2(A+B+C)} - 1 - 1 + e^{i(A+B+C)} + e^{i(A+B+C)} - 1$$

$$= e^{-i2\pi} - 1 - 1 + e^{i\pi} + e^{i\pi} - 1$$

$$= 1 - 1 - 1 - 1 - 1 - 1 = -4$$

$\therefore$ Absolute value of given determinant is 4.

31. (13) : Let E_1 : There is a gust of wind.

E_2 : There is no gust of wind.

A : Archer missed the target.

Now, $P(A) = P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)$

$$= \frac{3}{10} \cdot \frac{3}{5} + \frac{7}{10} \cdot \frac{3}{10} = \frac{39}{100}$$

$\therefore$ Required probability $= P(E_2/A) = \dfrac{P(E_2 \cap A)}{P(A)}$

$$= \frac{P(E_2) \cdot P(A/E_2)}{P(A)} = \frac{\frac{7}{10} \cdot \frac{3}{10}}{\frac{39}{100}} = \frac{7}{13} = \frac{7}{\lambda} \therefore \lambda = 13$$

32. (6) : We have, $\dfrac{(x-1)(x^2-1)(x^3-1)(x^4-1)}{((x-1)(x^2-1))^2}$

$$= \frac{(x^3-1)(x^4-1)}{(x-1)(x^2-1)} = (x^2 + x + 1)(x^2 + 1)$$

$$\therefore \lim_{x \to 1} \frac{(x-1)(x^2-1)(x^3-1)(x^4-1)}{((x-1)(x^2-1))^2}$$

$$= \lim_{x \to 1} (x^2 + x + 1)(x^2 + 1) = 6$$

33. (5) : $[\sin x]$ is discontinuous at $x = \pi/2, \pi, 2\pi$ and $[\cos x]$ is discontinuous at $x = 0, \pi/2, 3\pi/2, 2\pi$.

Thus, $f(x)$ is discontinuous at $x = 0, \dfrac{\pi}{2}, \pi, 3\pi/2, 2\pi$.

Thus, $f(x)$ is discontinuous and hence non-differentiable at 5 points.

34. (256) : $\int \sin x \cdot \cos x \cdot \cos 2x \cdot \cos 4x \cdot \cos 8x\, dx$

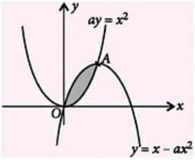
$$= \frac{1}{2}\int \sin 2x \cdot \cos 2x \cdot \cos 4x \cdot \cos 8x\, dx$$

$$= \frac{1}{4}\int \sin 4x \cdot \cos 4x \cdot \cos 8x\, dx$$

$$= \frac{1}{8}\int \sin 8x \cdot \cos 8x\, dx = \frac{1}{16}\int \sin 16x\, dx$$

$$= \frac{-1}{256}\cos 16x + C = \frac{-1}{\lambda}\cos 16x + c \therefore \lambda = 256$$

35. (1) : At point A,



$$\frac{x^2}{a} = x - ax^2$$

$$\Rightarrow x = 0,\ x = \frac{a}{1+a^2}$$

$\therefore$ Required area,

$$\Delta = \int_0^{a/1+a^2}\left(x - ax^2 - \frac{x^2}{a}\right)dx$$

$$\Rightarrow \frac{d\Delta}{da} = \int_0^{a/1+a^2}\left(-x^2 + \frac{x^2}{a^2}\right)dx + 0 \cdot \frac{d}{da}\left(\frac{a}{1+a^2}\right)$$

$$= \left[\frac{(1-a^2)}{3a^2}x^3\right]_0^{a/1+a^2} = \frac{a(1-a^2)}{3(1+a^2)^3} = \frac{a(1-a)(1+a)}{3(1+a^2)^3}$$

$$\Rightarrow \frac{d\Delta}{da} > 0\ \forall\ a \in (0,1) \text{ and } \frac{d\Delta}{da} < 0\ \forall\ a \in (1, \infty)$$

Thus, $a = 1$ is the point of maxima for Δ.

SOLUTION SENDERS

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36. (1) : Let the equation of side AB be

$$\frac{x}{a}+\frac{y}{b}=1 \quad \text{or} \quad bx+ay-ab=0$$

Centre and radius of circle are (2, 2) and 2 units respectively.

Since, AB touches the circles. Therefore,

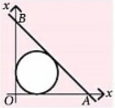
$$\frac{|2b+2a-ab|}{\sqrt{a^2+b^2}}=2$$

$$\Rightarrow\ -(2a+2b-ab)=2\sqrt{a^2+b^2}$$

('–' sign is used as (2, 2) lies below the line $bx + ay - ab = 0$)

If $P(h, k)$ is the midpoint of AB, then $a = 2h$, $b = 2k$

Thus, $4kh - 4h - 4k = 2\sqrt{4h^2+4k^2}$



Hence, locus of 'P' is,

$$x+y-xy+\sqrt{x^2+y^2}=0$$

$\therefore\ \lambda = 1$

37. (2) : Length of latus rectum $=\frac{2b^2}{a}$

For $xy = c^2$, $a = b$ and $\frac{a^2}{2}=c^2$

$\therefore$ Required length $=2\cdot\frac{a^2}{a}=2a=2\sqrt{2}c=2\sqrt{pc}\ \therefore\ p=2$

38. (2) : We have, $27^{\cos x} + 81^{\sin x}$

$$=3^{3\cos x}+3^{4\sin x}\geq 2\cdot\sqrt{3^{3\cos x}\cdot 3^{4\sin x}}$$

$$=2\cdot 3^{\frac{1}{2}(3\cos x+4\sin x)}\geq 2\cdot 3^{-\frac{5}{2}}=\frac{2}{9\sqrt{3}}=\frac{\lambda}{9\sqrt{3}}\ \therefore\ \lambda=2$$

39. (2) : $\angle A = \pi/2 \ \Rightarrow\ 2R = a$

Also, $r=(s-a)\tan\frac{A}{2}=(s-a)=\frac{1}{2}(b+c-a)$

$$\Rightarrow\ r+2R=\frac{1}{2}(a+b+c)=\frac{1}{p}(a+b+c)\ \therefore\ p=2$$

40. (2) : Clearly, the given curves intersect for $a \in R - \{0\}$.

For the parabola, $\frac{dy}{dx}=\frac{8}{y}$. For the ellipse, $\frac{dy}{dx}=\frac{-4x}{a^2y}$

For the curves to intersect orthogonally,

$$\left(\frac{8}{y}\right)\left(-\frac{4x}{a^2y}\right)=-1$$

$$\Rightarrow\ a^2y^2=32x\ \Rightarrow\ 16a^2x=32x\Rightarrow a^2=2$$

❖ ❖

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PRACTICE PAPER

Exam on 3rd July 2021

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PAPER-1

SINGLE OPTION CORRECT TYPE

1. Let S be the set of all complex numbers z satisfying $|z-2+i| \geq \sqrt{5}$. If the complex number z_0 is such that $\dfrac{1}{|z_0-1|}$ is the maximum of the set $\left\{\dfrac{1}{|z-1|} : z \in S\right\}$, then the principal argument of $\dfrac{4-z_0-\bar{z}_0}{z_0-\bar{z}_0+2i}$ is

(a) $\dfrac{\pi}{2}$ (b) $\dfrac{\pi}{4}$ (c) $-\dfrac{\pi}{2}$ (d) $\dfrac{3\pi}{4}$

2. Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be two non-constant differentiable functions. If $f'(x) = (e^{(f(x)\,-\,g(x))})g'(x)$ for all $x \in \mathbb{R}$, and $f(1) = g(2) = 1$, then which of the following statement is TRUE?

(a) $f(2) < 1 - \log_e 2$ (b) $f(2) > 1 - \log_e 2$

(c) $g(2) > 1 - \log_e 2$ (d) $g(2) < 1 - \log_e 2$

3. Area of the region

$\left\{(x, y) \in R^2 : y \geq \sqrt{|x+3|}, 5y \leq x+9 \leq 15\right\}$ is equal to

(a) 1/6 (b) 4/3 (c) 3/2 (d) 5/3

4. The line passing through the extremity A of the major axis and extremity B of the minor axis of the ellipse $x^2 + 9y^2 = 9$ meets the auxiliary circle at the point M. The area of the triangle AMO is

(a) 31/10 (b) 29/10 (c) 21/10 (d) 27/10

5. An urn contains m white and n black balls. A ball is drawn at random and is put back into the urn along with k additional balls of the same colour as that of the ball drawn. If now a ball is drawn, then the probability, that it is white, is

(a) $\dfrac{m+k}{m+n+k}$ (b) $\dfrac{n+k}{m+n+k}$

(c) $\dfrac{m}{m+n+k}$ (d) $\dfrac{m}{m+n}$

6. Let $P(x) = a_0 + a_1x^2 + a_2x^4 + \ldots + a_nx^{2n}$ with all $a_i > 0$, $i = 0, 1, 2, \ldots, n$. Then $P(x)$ has

(a) neither maximum nor minimum

(b) only one maximum

(c) only one minimum

(d) one maximum and one minimum

ONE OR MORE THAN ONE OPTION(S) CORRECT TYPE

7. Let $f : \left[-\dfrac{1}{2}, 2\right] \to R$ and $g : \left[-\dfrac{1}{2}, 2\right] \to R$ be functions defined by $f(x) = [x^2] - 3$ and $g(x) = |x|\, f(x) + |4x - 7|\, f(x)$, where $[y]$ denotes the greatest integer less than or equal to y for $y \in R$. Then

(a) f is discontinuous exactly at three points in $\left[-\dfrac{1}{2}, 2\right]$

(b) f is discontinuous exactly at four points in $\left[-\dfrac{1}{2}, 2\right]$

(c) g is not differentiable exactly at four points in $\left(-\dfrac{1}{2}, 2\right)$

(d) g is not differentiable exactly at five points in $\left(-\dfrac{1}{2}, 2\right)$

8. Let X and Y be two arbitrary, 3×3 non-zero, skew-symmetric matrices and Z be an arbitrary 3×3 non-zero symmetric matrix. Then which of the following matrices is (are) skew symmetric?

(a) $Y^3Z^4 - Z^4Y^3$ (b) $X^{44} + Y^{44}$

(c) $X^4Z^3 - Z^3X^4$ (d) $X^{23} + Y^{23}$

9. Let a, b, x and y be real numbers such that $a - b = 1$ and $y \neq 0$. If the complex number $z = x + iy$ satisfies $\text{Im}\left(\dfrac{az+b}{z+1}\right) = y$, then which of the following is (are) possible value (s) of x?

(a) $1+\sqrt{1+y^2}$ (b) $-1+\sqrt{1-y^2}$

(c) $1-\sqrt{1+y^2}$ (d) $-1-\sqrt{1-y^2}$

10. Let ΔPQR be a triangle.

Let $\vec{a}=\overrightarrow{QR}, \vec{b}=\overrightarrow{RP}$ and $\vec{c}=\overrightarrow{PQ}$.

If $|\vec{a}|=12$, $|\vec{b}|=4\sqrt{3}$ and $\vec{b}\cdot\vec{c}=24$, then which of the following is (are) true?

(a) $\dfrac{|\vec{c}|^2}{2}-|\vec{a}|=12$ (b) $\dfrac{|\vec{c}|^2}{2}+|\vec{a}|=30$

(c) $|\vec{a}\times\vec{b}+\vec{c}\times\vec{a}|=48\sqrt{3}$ (d) $\vec{a}\cdot\vec{b}=-72$

11. The option(s) with the values of a and L that satisfy the following equation is(are)

$$\frac{\int_0^{4\pi} e^t(\sin^6 at+\cos^4 at)dt}{\int_0^{\pi} e^t(\sin^6 at+\cos^4 at)dt}=L?$$

(a) $a=2,\ L=\dfrac{e^{4\pi}-1}{e^{\pi}-1}$ (b) $a=2,\ L=\dfrac{e^{4\pi}+1}{e^{\pi}+1}$

(c) $a=4,\ L=\dfrac{e^{4\pi}-1}{e^{\pi}-1}$ (d) $a=4,\ L=\dfrac{e^{4\pi}+1}{e^{\pi}+1}$

12. Consider two straight lines, each of which is tangent to both the circle $x^2+y^2=\frac{1}{2}$ and the parabola $y^2=4x$. Let these lines intersect at the point Q. Consider the ellipse whose center is at the origin $O(0, 0)$ and whose semi-major axis is OQ. If the length of the minor axis of this ellipse is $\sqrt{2}$, then which of the following statement(s) is (are) TRUE?

(a) For the ellipse, the eccentricity is $\frac{1}{\sqrt{2}}$ and the length of the latus rectum is 1

(b) For the ellipse, the eccentricity is $\frac{1}{2}$ and the length of the latus rectum is $\frac{1}{2}$

(c) The area of the region bounded by the ellipse between the lines $x=\frac{1}{\sqrt{2}}$ and $x=1$ is $\frac{1}{4\sqrt{2}}(\pi-2)$

(d) The area of the region bounded by the ellipse between the lines $x=\frac{1}{\sqrt{2}}$ and $x=1$ is $\frac{1}{16}(\pi-2)$

NUMERICAL VALUE TYPE

13. A cylindrical container is to be made from certain solid material with the following constraints : It has a fixed inner volume of V mm^3, has a 2 mm thick solid wall and is open at the top. The bottom of the container is a solid circular disc of thickness 2 mm and is of radius equal to the outer radius of the container. If the volume of the material used to make the container is minimum when the inner radius of the container is 10 mm, then the value of $\frac{V}{250\pi}$ is _____.

14. Suppose that all the terms of an arithmetic progression (A.P.) are natural numbers. If the ratio of the sum of the first seven terms to the sum of the first eleven terms is 6 : 11 and the seventh term lies in between 130 and 140, then the common difference of this A.P. is _____.

15. Let $f: R\to R$ be a continuous odd function, which vanishes exactly at one point and $f(1)=\frac{1}{2}$. Suppose that $F(x)=\int_{-1}^{x} f(t)dt$ for all $x\in[-1, 2]$ and $G(x)=\int_{-1}^{x} t\,|f(f(t))|\,dt$ for all $x\in[-1, 2]$. If $\lim_{x\to 1}\frac{F(x)}{G(x)}=\frac{1}{14}$, then the value of $f\left(\frac{1}{2}\right)$ is _____.

16. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that $\vec{a}\cdot\vec{b}=0$. For some $x, y\in\mathbb{R}$, let $\vec{c}=x\vec{a}+y\vec{b}+(\vec{a}\times\vec{b})$. If $|\vec{c}|=2$ and the vector $\vec{c}$ is inclined at the same angle α to both $\vec{a}$ and $\vec{b}$, then the value of $8\cos^2\alpha$ is _____.

17. Let $X=({}^{10}C_1)^2+2({}^{10}C_2)^2+3({}^{10}C_3)^2+\ldots+10({}^{10}C_{10})^2$, where ${}^{10}C_r$, $r\in\{1, 2, \ldots, 10\}$ denote binomial coefficients. Then, the value of $\frac{1}{1430}X$ is _____.

18. Let $\alpha, \beta\in R$ be such that $\lim_{x\to 0}\frac{x^2\sin(\beta x)}{\alpha x-\sin x}=1$. Then $6(\alpha+\beta)$ equals _____.

······

Monthly Test Drive CLASS XII ANSWER KEY

1. (b) 2. (a) 3. (a) 4. (a) 5. (c)
6. (a) 7. (a,b) 8. (b,c) 9. (b) 10. (a,c)
11. (a,d) 12. (a,d) 13. (b) 14. (a) 15. (c)
16. (c) 17. (0) 18. (9) 19. (6) 20. (2)

PAPER-2

SINGLE DIGIT INTEGER ANSWER TYPE

1. If for any integer k, $\alpha_k = \cos\left(\frac{k\pi}{7}\right) + i\sin\left(\frac{k\pi}{7}\right)$ where $i = \sqrt{-1}$. Then the value of the expression $\dfrac{\sum_{k=1}^{12} |\alpha_{k+1} - \alpha_k|}{\sum_{k=1}^{3} |\alpha_{4k-1} - \alpha_{4k-2}|}$ is ______.

2. The number of points of discontinuity of the function $f(x) = \left[\frac{6x}{\pi}\right]\cos\left[\frac{3x}{\pi}\right]$, (where $[y]$ denotes the largest integer less than or equal to y) in the interval $\left(\frac{\pi}{10}, \frac{11\pi}{10}\right)$, is ______.

3. Let $f(n)$ denote the square of the sum of the digits of natural number n, where $f^2(n)$ denote $f(f(n))$, $f^3(n)$ denote $f(f(f(n)))$ and so on. Then value of $\dfrac{f^{2011}(2011) - f^{2010}(2011)}{f^{2013}(2011) - f^{2012}(2011)} =$ ______.

4. The value of $\Delta = \begin{vmatrix} 1 & 1+ac & 1+bc \\ 1 & 1+ad & 1+bd \\ 1 & 1+ae & 1+be \end{vmatrix} =$ ______.

5. For each positive integer n, let

$$y_n = \frac{1}{n}((n+1)(n+2)\ldots(n+n))^{\frac{1}{n}}$$

For $x \in \mathbb{R}$, let $[x]$ be the greatest integer less than or equal to x. If $\lim_{n\to\infty} y_n = L$, then the value of $[L]$ is ____.

6. Let $f: R \to R$ be a differentiable function such that $f(0) = 0, f\left(\frac{\pi}{2}\right) = 3$ and $f'(0) = 1$.

If $g(x) = \int_x^{\pi/2} [f'(t)\operatorname{cosec} t - \cot t \operatorname{cosec} t\, f(t)]dt$

for $x \in \left(0, \frac{\pi}{2}\right]$, then $\lim_{x\to 0} g(x) =$ ______.

ONE OR MORE THAN ONE OPTION(S) CORRECT TYPE

7. Let $f(x) = (x - a)^2 g(x)$, $g(a) \neq 0$, Then $f(x)$ has

(a) minimum if $g(a) > 0$

(b) minimum if $g(a) < 0$

(c) maximum if $g(a) > 0$

(d) maximum if $g(a) < 0$

8. If the foci of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ coincide with the foci of $\frac{x^2}{25} + \frac{y^2}{9} = 1$ and eccentricity of the hyperbola is 2, then

(a) $a^2 + b^2 = 16$

(b) there is no director circle of the hyperbola

(c) centre of the director circle is $(0, 0)$

(d) length of latus rectum of the hyperbola = 12

9. If N is the number of ways of dividing $2n$ people into n couples, then

(a) $2^n N = (2n)!$

(b) $N(n!) = 1\cdot3\cdot5 \ldots (2n - 1)$

(c) $N = (1 \times 3 \times \ldots \times (2n - 1))(1 \times 2 \times \ldots \times n)$

(d) none of these

10. A non-zero vector $\vec{a}$ is parallel to the line of intersection of the plane determined by the vectors, $\hat{i}, \hat{i} + \hat{j}$ and the plane determined by the vectors $\hat{i} - \hat{j}, \hat{i} + \hat{k}$. The angle between $\vec{a}$ and the vector $\hat{i} - 2\hat{j} + 2\hat{k}$ is

(a) $\pi/4$ (b) $3\pi/4$ (c) $\pi/3$ (d) $2\pi/3$

11. If $y = f(x)$ is the solution of equation $ydx + dy$ $-e^x y^2 dy$, $f(0) = 1$ and area bounded by the curve $y = f(x)$, $y = e^x$ and $x = 1$ is A, then

(a) curve $y = f(x)$ is passing through $(-2, e)$

(b) curve $y = f(x)$ is passing through $(1, 1/e)$

(c) curve $y = f(x)$ is passing through $(1, 1/3)$

(d) $A = e + \frac{2}{\sqrt{e}} - 3$

12. The plane $ax + by = 0$ is rotated through an angle α about its line of intersection with the plane $z = 0$. Then the equation to the plane in new position is/are

(a) $ax + by + z\sqrt{a^2 + b^2}\tan\alpha = 0$

(b) $ax + by - z\sqrt{a^2 + b^2}\tan\alpha = 0$

(c) $ax + by + z\sqrt{a^2 + b^2}\cot\alpha = 0$

(d) $ax + by - z\sqrt{a^2 + b^2}\cot\alpha = 0$

NUMERICAL VALUE TYPE

13. Let S be the sample space of all 3×3 matrices with entries from the set $\{0, 1\}$. Let the events E_1 and E_2 be

given by $E_1 = \{A \in S : \text{Det } A = 0\}$ and $E_2 = \{A \in S : \text{Sum of entries of } A \text{ is } 7\}$.
If a matrix is chosen at random from S, then the conditional probability $P(E_1/E_2)$ equals ______.

14. Let the point B be the reflection of the point $A(2, 3)$ with respect to the line $8x - 6y - 23 = 0$. Let Γ_A and Γ_B be circles of radii 2 and 1 with centres A and B respectively. Let T be a common tangent to the circles Γ_A and Γ_B such that both the circles are on the same side of T. If C is the point of intersection of T and the line passing through A and B, then the length of the line segment AC is ______.

15. If $f(x) = (x - a)(x - b)$ for $a, b \in R$, then minimum number of roots of equation $\pi(f'(x))^2 \cos(\pi f(x)) + \sin(\pi f(x)) f''(x) = 0$ in (α, β), where $f(\alpha) = f(\beta) = 0$, is (here $\alpha < a < b < \beta$) ______.

16. A sample of size 4 is drawn with replacement (without replacement) from an urn containing 12 balls, of which 8 are white. The conditional probability that the ball drawn on the third draw was white, given that the sample contains 3 white balls is $\frac{3}{s}$, where s is equal to ______.

17. If the largest $x \in N$ such that ${}^{x-1}C_4 - {}^{x-1}C_3 < \frac{5}{4}({}^{x-2}P_2)$, then the sum of digits of x is ______.

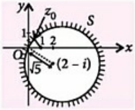
18. Let $a_1, a_2, a_3, \ldots\ldots, a_{11}$ be real numbers satisfying $a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for $k = 3, 4, \ldots, 11$. If $\frac{a_1^2 + a_2^2 + \ldots\ldots + a_{11}^2}{11} = 90$, then the value of $\frac{a_1 + a_2 + \ldots\ldots + a_{11}}{11}$ is ______.

SOLUTIONS

PAPER-1

1. (c) : $\frac{1}{|z-1|}$, $z \in S$, attains its maximum when $|z - 1|$ attains its minimum.

Now the curve $|z - 2 + i| \geq \sqrt{5}$ is shown in figure.



Consider $\frac{4 - z_0 - \bar{z}_0}{z_0 - \bar{z}_0 + 2i}$

$$= \frac{4 - 2\,\text{Re}\,z_0}{2i\,\text{Im}\,z_0 + 2i} = \frac{2(2 - \text{Re}\,z_0)}{2i(\text{Im}\,z_0 + 1)}$$

$= -i\lambda$, where $\lambda > 0$

Since $2 - \text{Re}\,z_0 > 0$ and $\text{Im}\,z_0 > 0$

$\therefore$ The principal argument of $\frac{4 - z_0 - \bar{z}_0}{z_0 - \bar{z}_0 + 2i}$ is $-\pi/2$.

2. (b) : We have, $f'(x) = e^{(f(x) - g(x))}\, g'(x)$

$\Rightarrow\ e^{-f(x)} f'(x) = e^{-g(x)} g'(x)$

Integrating we get, $-e^{-f(x)} = -e^{-g(x)} + \lambda$

i.e., $e^{-g(x)} - e^{-f(x)} = \lambda$

Putting $x = 1$ and $x = 2$ respectively, we get

$$e^{-g(1)} - e^{-f(1)} = \lambda \text{ and } e^{-g(2)} - e^{-f(2)} = \lambda$$

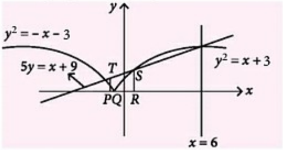
We have, $e^{-g(1)} - e^{-f(1)} = e^{-g(2)} - e^{-f(2)}$

$\Rightarrow\ e^{-g(1)} - e^{-1} = e^{-1} - e^{-f(2)} \Rightarrow e^{-g(1)} + e^{-f(2)} = 2/e$

$\Rightarrow\ e^{-f(2)} < \frac{2}{e}\ \Rightarrow -f(2) < \log_e 2 - 1$

$\therefore\ f(2) > 1 - \log_e 2$

3. (c) : The graph of the region is sketched below.



Note that $y \geq \sqrt{|x + 3|}$ means $y^2 \geq |x + 3|$

i.e. $y^2 \geq x + 3$ or $y^2 \geq -x - 3$

$P \equiv (-4, 0)$, $Q(-3, 0)$, $R(1, 0)$, $S(1, 2)$, $T(-4, 1)$

$$[PQT] = \int_{-4}^{-3} \sqrt{-x-3}\, dx = \int_0^1 \sqrt{t}\, dt = \left[\frac{t^{3/2}}{3/2}\right]_0^1 = \frac{2}{3}$$

$$\text{Again, } [QRS] = \int_{-3}^{1} \sqrt{x+3}\,dx = \int_0^4 \sqrt{u}\, du = \left[\frac{2}{3}u^{3/2}\right]_0^4 = \frac{16}{3}$$

$$[PRST] = \frac{1}{2}(1+2)\cdot 5 = \frac{15}{2}$$

$\therefore$ The desired area $= \frac{15}{2} - \frac{16}{3} - \frac{2}{3} = \frac{3}{2}$

4. (d)

5. (d) :

Sequence	Probability
WW	$\frac{m}{m+n} \cdot \frac{m+k}{m+n+k}$
BW	$\frac{n}{m+n} \cdot \frac{m}{m+n+k}$

$\therefore$ Required probability $= \frac{m}{m+n}\left[\frac{m+n+k}{m+n+k}\right] = \frac{m}{m+n}$

6. (c) : $P(x) > 0$

$P'(x) = 2a_1x + 4a_2x^3 + \ldots = 2x(a_1 + 2a_2x^2 + \ldots)$

$P'(x) = 0 \Rightarrow x = 0$, since the second factor is positive.

$\therefore$ $P(x)$ has minimum at $x = 0$ and $P_{\min} = a_0$

7. (b, c) : $f(x) = [x^2] - 3$ is discontinuous at

$x = 1, \sqrt{2}, \sqrt{3}, 2$ in $\left[-\frac{1}{2}, 2\right]$

Also, $g(x) = (|x| + |4x - 7|)([x^2] - 3)$

As at these points $|x| + |4x - 7|$ doesn't vanish. Thus g is discontinuous at $x = 1, \sqrt{2}, \sqrt{3}$ in $\left(-\frac{1}{2}, 2\right)$

Hence, g is not differentiable as well.

In the neighbourhood of 0

$g(x) = (|x| + |4x - 7|)(-3)$

Thus g is non-differentiable at $x = 0$.

Similarly, g is not differentiable at exactly four points.

8. (c, d) : $X^T = -X, Y^T = -Y, Z^T = Z$

Now, $(Y^3Z^4 - Z^4Y^3)^T = (Y^3Z^4)^T - (Z^4Y^3)^T$
$= -Z^4Y^3 + Y^3Z^4 = Y^3Z^4 - Z^4Y^3$

Hence, symmetric.

Similarly $(X^{44} + Y^{44})^T = X^{44} + Y^{44}$, so symmetric.

$(X^4Z^3 - Z^3X^4)^T = (X^4Z^3)^T - (Z^3X^4)^T$
$= Z^3X^4 - X^4Z^3 = -(X^4Z^3 - Z^3X^4)$

Hence, skew symmetric.

Also, $(X^{23} + Y^{23})^T = -X^{23} - Y^{23}$
$= -(X^{23} + Y^{23})$, so skew symmetric.

9. (b, d) : $\operatorname{Im}\left(\frac{a(x+iy)+b}{x+iy+1}\right) = y$

$$\Rightarrow \operatorname{Im}\left(\frac{(ax+b)+iay}{x+1+iy}\right) = y$$

$$\Rightarrow \operatorname{Im}\left(\frac{[(ax+b)+iay][(x+1)-iy]}{(x+1)^2+y^2}\right) = y$$

$$\Rightarrow \operatorname{Im}\left(\frac{(ax+b)(x+1)+ay^2}{(x+1)^2+y^2} + i\frac{ay(x+1)-y(ax+b)}{(x+1)^2+y^2}\right) = y$$

We then have,

$$\frac{ay-by}{(x+1)^2+y^2} = y \Rightarrow y = y\{(x+1)^2+y^2\}$$

As $y \neq 0$, we have $(x+1)^2 + y^2 = 1$

$$\Rightarrow x+1 = \pm\sqrt{1-y^2}$$

$$\therefore x = -1+\sqrt{1-y^2} \text{ or } -1-\sqrt{1-y^2}$$

10. (a, c, d) : As $\vec{a}+\vec{b}+\vec{c} = 0$

$\Rightarrow |\vec{a}|^2 = |\vec{b}|^2 + |\vec{c}|^2 + 2\vec{b}\cdot\vec{c}$

$\Rightarrow 144 = 48 + |\vec{c}|^2 + 48 \Rightarrow |\vec{c}|^2 = 48$

Again, $|\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b}$

$$\Rightarrow \vec{a}\cdot\vec{b} = \frac{48-144-48}{2} = -72$$

Also, $|\vec{a}\times\vec{b} + \vec{c}\times\vec{a}| = |\vec{a}\times\vec{b} + \vec{a}\times\vec{b}| = 2|\vec{a}\times\vec{b}|$

$$= 2\sqrt{|\vec{a}|^2|\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2} = 2\sqrt{12^2\cdot 48 - (-72)^2}$$
$$= 2\cdot 12\sqrt{48-36} = 2\cdot 12\cdot 2\sqrt{3} = 48\sqrt{3}$$

11. (a, c) : $I_1 = \int_0^{4\pi} e^t(\sin^6 at + \cos^4 at)dt$

$$= \sum_{k=0}^{3}\int_{k\pi}^{(k+1)\pi} e^t(\sin^6 at + \cos^4 at)dt$$

$$= \sum_{k=0}^{3}\int_0^{\pi} e^{(k\pi+t)}(\sin^6 at + \cos^4 at)dt$$

$$= \left(\sum_{k=0}^{3} e^{k\pi}\right)\int_0^{\pi} e^t(\sin^6 at + \cos^4 at)dt$$

$$= (1+e^{\pi}+e^{2\pi}+e^{3\pi})\int_0^{\pi} e^t(\sin^6 at + \cos^4 at)dt$$

$\therefore$ The given expression

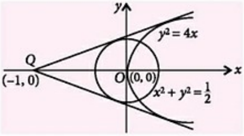
$$= 1+e^{\pi}+e^{2\pi}+e^{3\pi} = \frac{e^{4\pi}-1}{e^{\pi}-1}$$

Also, $a = 2$ and $a = 4$ both cases hold.

12. (a, c) : The equation of common tangent be $y = px + 1/p$.

We have, $\frac{|1/p|}{\sqrt{1+p^2}} = \frac{1}{\sqrt{2}} \Rightarrow p^4 + p^2 - 2 = 0$

$\Rightarrow (p^2+2)(p^2-1) = 0 \Rightarrow p^2 = 1$ ($\because p^2 \geq 0$)



$\therefore$ The equations of common tangents are $x + y + 1 = 0$ and $x - y + 1 = 0$

So, the point Q is $(-1, 0)$.

The equation of ellipse is $\frac{x^2}{1}+\frac{y^2}{1/2}=1$

The eccentricity of the ellipse = $\sqrt{1-\frac{1}{2}}=\frac{1}{\sqrt{2}}$

Length of latus rectum = $\frac{2b^2}{a}=\frac{2\cdot\frac{1}{2}}{1}=1$

The required area = $2\int_{1/\sqrt{2}}^{1}\frac{1}{\sqrt{2}}\sqrt{1-x^2}\,dx$

$$=\sqrt{2}\left[\left(\frac{x}{2}\right)\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}x\right]_{1/\sqrt{2}}^{1}=\frac{\pi-2}{4\sqrt{2}}$$

13. (4)

14. (9) : $\frac{S_7}{S_{11}}=\frac{6}{11} \Rightarrow \frac{7(2a+6d)}{11(2a+10d)}=\frac{6}{11}$

$\Rightarrow 7a+21d=6a+30d \Rightarrow a=9d$

Now, $130 < a+6d < 140$

i.e., $130 < 15d < 140$. Hence $d=9$

15. (7) : $\lim\limits_{x\to1}\frac{F(x)}{G(x)}=\lim\limits_{x\to1}\frac{F'(x)}{G'(x)}=\lim\limits_{x\to1}\frac{f(x)}{x\,|f(f(x))|}=\frac{1}{14}$

$$=\frac{f(1)}{\left|f\left(\frac{1}{2}\right)\right|}=\frac{1}{14}$$

Thus, $\left|f\left(\frac{1}{2}\right)\right|=7$. But $f\left(\frac{1}{2}\right)\neq-7$ as $f(x)$ vanishes exactly at one point.

16. (3) : Here $\vec{c}=x\vec{a}+y\vec{b}+(\vec{a}\times\vec{b})$

Also, $\vec{a}\cdot\vec{c}=\vec{b}\cdot\vec{c}=2\cos\alpha$

We have, $\vec{c}\cdot\vec{a}=x\vec{a}\cdot\vec{a}+0+0$

$\therefore\ x=\vec{c}\cdot\vec{a}=2\cos\alpha$ Similarly, $y=2\cos\alpha$

Now, $c^2=x^2+y^2+1$

$\Rightarrow 4=2(4\cos^2\alpha)+1 \Rightarrow 3=8\cos^2\alpha$

17. (646) : $X=\sum\limits_{r=1}^{10}r({}^{10}C_r)^2=\sum\limits_{r=1}^{10}r\cdot{}^{10}C_r\cdot{}^{10}C_r$

$$=10\cdot\sum_{r=1}^{10}{}^{9}C_{r-1}\cdot{}^{10}C_{10-r}=10\cdot{}^{19}C_9$$

$$\therefore\ \frac{X}{1430}=\frac{10\cdot{}^{19}C_9}{1430}=\frac{{}^{19}C_9}{143}=646$$

18. (7) : Given, $\lim\limits_{x\to0}\frac{x^2\sin(\beta x)}{\alpha x-\sin x}=1$

If $\alpha\neq1$, then $\lim\limits_{x\to0}\frac{x^2\sin(\beta x)}{\alpha x-\sin x}=\lim\limits_{x\to0}\frac{x\sin(\beta x)}{\alpha-\frac{\sin x}{x}}=0$

$\therefore\ \alpha=1$

Then $\lim\limits_{x\to0}\frac{x^2\sin(\beta x)}{x-\sin x}=1 \Rightarrow \lim\limits_{x\to0}\frac{\beta x^3\cdot\frac{\sin(\beta x)}{\beta x}}{x^3\left(\frac{x-\sin x}{x^3}\right)}=1$

$$\left[\because\ \lim_{x\to0}\frac{x-\sin x}{x^3}=\frac{1-\cos x}{3x^2}=\frac{\sin x}{6x}=\frac{1}{6}\right]$$

$\Rightarrow 6\beta=1$. Thus, $6\alpha+6\beta=7$

PAPER-2

1. (4) : $\alpha_k=\cos\frac{2k\pi}{14}+i\sin\frac{2k\pi}{14}=e^{i\frac{2\pi k}{14}}$

$$\therefore\ \frac{\sum\limits_{k=1}^{12}\left|e^{i2(k+1)\frac{\pi}{14}}-e^{i\frac{2k\pi}{14}}\right|}{\sum\limits_{k=1}^{3}\left|e^{\frac{i2(4k-1)\pi}{14}}-e^{i\frac{2(4k-2)\pi}{14}}\right|}=\frac{\sum\limits_{k=1}^{12}\left|e^{i\frac{2\pi}{14}}-1\right|}{\sum\limits_{k=1}^{3}\left|e^{i\frac{2\pi}{14}}-1\right|}=\frac{12}{3}=4$$

2. (4) : $f(x)=\left[\frac{6x}{\pi}\right]\cos\left[\frac{3x}{\pi}\right]$

It is discontinuous when either $\frac{6x}{\pi}\in I$ or $\frac{3x}{\pi}\in I$

i.e., when $x=\pi/6$ or $\pi/3$ or $\pi/2$ or π

3. (1) : $f(2011)=(2+0+1+1)^2=16$

$f^2(2011)=(1+6)^2=49$; $f^3(2011)=(4+9)^2=169$

$f^4(2011)=(1+6+9)^2=256$

$f^5(2011)=(2+5+6)^2=169$

It is a periodic function.

$\therefore\ f^{2n}(2011)=256$ and $f^{2n+1}(2011)=169$

4. (0)

5. (1) : We have,

$$\log y_n=\frac{1}{n}\left\{\log\left(1+\frac{1}{n}\right)+\log\left(1+\frac{2}{n}\right)+...+\log\left(1+\frac{n}{n}\right)\right\}$$

$$\therefore\ \lim_{n\to\infty}\log y_n=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\log\left(1+\frac{r}{n}\right)$$

$$=\int_0^1\log(1+x)dx=\int_1^2\log x\,dx$$

$$=\left[x\log x-x\right]_1^2=2\log2-1=\log\frac{4}{e}$$

$\therefore\ L=4/e$ Thus, $[L]=1$.

6. (2) : Let $g(x)$

$$= \int_x^{\pi/2} [f'(t)\operatorname{cosec} t - \cot t \operatorname{cosec} t\, f(t)]\,dt$$

$$= \int_x^{\pi/2} \frac{d}{dt}(f(t)\operatorname{cosec} t))$$

So, $g(x) = f(\pi/2)\operatorname{cosec}\frac{\pi}{2} - f(x)\operatorname{cosec} x$

$= 3 - f(x)\operatorname{cosec} x$

$\therefore \quad g(x) = 3 - \dfrac{f(x)}{\sin x}$

$$\lim_{x\to 0} g(x) = 3 - \lim_{x\to 0}\frac{f(x)}{\sin x}$$

As the above is a 0/0 form, use L'Hospital's rule to get

$$\lim_{x\to 0} g(x) = 3 - \lim_{x\to 0}\frac{f'(x)}{\cos x} = 3 - f'(0) = 3 - 1 = 2$$

7. (a, d) : $f'(x) = 2(x - a)\,g(x) + (x - a)^2 g'(x)$

$= (x - a)[2g(x) + (x - a)\,g'(x)]$

$f'(x) = 0$ when $x = a$

$f''(x) = 2g(x) + 4(x - a)\,g'(x) + (x - a)^2 g''(x)$

$\Rightarrow \quad f''(a) = 2g(a)$

$g(a) > 0 \Rightarrow f''(a) > 0$ and $f(x)$ has minimum at $x = a$.

$g(a) < 0 \Rightarrow f''(a) < 0$ and $f(x)$ has maximum at $x = a$.

8. (a, b, d) : For the ellipse, $a = 5$ and $e = \sqrt{\dfrac{25-9}{25}} = \dfrac{4}{5}$

$\therefore \quad ae = 4$

Hence, the foci are $(-4, 0)$ and $(4, 0)$.

For the hyperbola, $ae = 4$, $e = 2$

$\therefore \quad a = 2$

$b^2 = 4(4 - 1) = 12$

$\Rightarrow \quad b = \sqrt{12}$

$\therefore$ Length of latus rectum $= \dfrac{2b^2}{a} = 2 \times \dfrac{12}{2} = 12$

9. (a, c) : $N = ({}^{2n}C_2)\,({}^{2n-2}C_2)\ldots({}^{4}C_2)\,({}^{2}C_2) = \dfrac{(2n)!}{2^n}$

$= \dfrac{(1)(2)(3)\ldots(2n-1)(2n)}{2^n}$

$= (1)\,(3)\,\ldots\,(2n - 1)\,(1 \times 2 \times \ldots \times n)$

10. (a, b) : A vector normal to the plane containing vectors $\hat{i}$ and $\hat{i} + \hat{j}$ is

$$\vec{p} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{vmatrix} = \hat{k}$$

A vector normal to the plane containing vectors $\hat{i} - \hat{j}, \hat{i} + \hat{k}$ is,

$$\vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = -\hat{i} - \hat{j} + \hat{k}.$$

Vector $\vec{a}$ is parallel to vector $\vec{p} \times \vec{q}$.

$$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ -1 & -1 & 1 \end{vmatrix} = \hat{i} - \hat{j}$$

Therefore, a vector in direction of $\vec{a}$ is $\hat{i} - \hat{j}$.

Now if θ is the angle between $\vec{a}$ and $\hat{i} - 2\hat{j} + 2\hat{k}$, then

$$\cos\theta = \pm\frac{1\cdot 1 + (-1)\cdot(-2)}{\sqrt{1+1}\,\sqrt{1+4+4}} = \pm\frac{3}{\sqrt{2}\cdot 3}$$

$$\Rightarrow \quad \cos\theta = \pm\frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4} \text{ or } \frac{3\pi}{4}$$

11. (a, d) : $ydx + dy = -e^x y^2 dy$

$$\Rightarrow \quad \frac{-(e^{-x}ydx + e^{-x}dy)}{y^2} = dy \;\Rightarrow\; d\left(\frac{e^{-x}}{y}\right) = dy$$

$\Rightarrow \quad e^{-x} = y^2 + cy \quad \because f(0) = 1, c = 0$

$\Rightarrow \quad e^{-x} = y^2 \Rightarrow y = e^{-x/2}$

$$\Rightarrow \quad A = \int_0^1 (e^x - e^{-x/2})\,dx = e + \frac{2}{\sqrt{e}} - 3$$

At $x = -2$, $y = e$

$\therefore$ Curve $y(x)$ passes through $(-2, e)$.

12. (a, b) : Given planes are $ax + by = 0$ ...(i)

and $z = 0$ ...(ii)

$\therefore$ Equation of any plane passing through the line of intersection of planes (i) and (ii) may be taken as

$ax + by + kz = 0$ ...(iii)

The direction cosines of a normal to the plane (iii) are

$$\frac{a}{\sqrt{a^2+b^2+k^2}}, \frac{b}{\sqrt{a^2+b^2+k^2}}, \frac{k}{\sqrt{a^2+b^2+k^2}}$$

The direction cosines of a normal to the plane (i) are

$$\frac{a}{\sqrt{a^2+b^2}}, \frac{b}{\sqrt{a^2+b^2}}, 0$$

Since the angle between the planes (i) and (ii) is α

$$\therefore \quad \cos\alpha = \frac{a\cdot a + b\cdot b + k\cdot 0}{\sqrt{a^2+b^2+k^2}\,\sqrt{a^2+b^2}} = \sqrt{\frac{a^2+b^2}{a^2+b^2+k^2}}$$

$\Rightarrow \quad k^2\cos^2\alpha = a^2(1 - \cos^2\alpha) + b^2(1 - \cos^2\alpha)$

$$\Rightarrow \quad k^2 = \frac{(a^2+b^2)\sin^2\alpha}{\cos^2\alpha} \Rightarrow k = \pm\sqrt{a^2+b^2}\tan\alpha$$

Putting in (iii), we get equation of plane as;

$$ax + by \pm z\sqrt{a^2 + b^2}\tan\alpha = 0$$

13. (0.5) : Given, E_2 : Sum of entries of $A = 7$

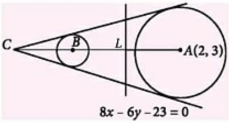
$\therefore \quad n(E_2) = {}^9C_2 = 36$

Now, $E_1 \cap E_2$ will be the set of those matrices, where both zeros lie in same row or in the same column.

$\therefore \quad n(E_1 \cap E_2) = 2 \times {}^3C_2 \times {}^3C_2 = 18$

Now, $P(E_1 / E_2) = \dfrac{n(E_1 \cap E_2)}{n(E_2)} = \dfrac{18}{36} = \dfrac{1}{2} = 0.5$

14. (10) :



Distance of A from $L = \dfrac{|8 \cdot 2 - 6 \cdot 3 - 23|}{\sqrt{8^2 + 6^2}}$

$= \dfrac{|16 - 18 - 23|}{10} = \dfrac{25}{10} = \dfrac{5}{2}$

From similar triangles, $\dfrac{CA}{CB} = \dfrac{2}{1} \Rightarrow \dfrac{CA}{AC - 5} = \dfrac{2}{1}$

$\Rightarrow \quad AC = 10$

15. (4) : Let $g(x) = f'(x)\sin(\pi f(x))$

$g(a) = f'(a)\sin(\pi f(a)) = f'(a)\sin 0 = 0$

Similarly $g(b) = 0$

$\Rightarrow \quad f'\left(\dfrac{a+b}{2}\right) = 0 \;\therefore\; g\left(\dfrac{a+b}{2}\right) = 0$

Also $f(\alpha) = f(\beta) = 0$

So, according to Rolle's theorem,

$g'(x) = 0$ has at least one root in (α, a), $\left(a, \dfrac{a+b}{2}\right)$, $\left(\dfrac{a+b}{2}, b\right)$ and (b, β).

i.e., Equation has minimum 4 roots.

16. (4) : Let A denote the event that the sample contains exactly three white balls and let B be the event that the ball drawn on the third draw was white.

Now, $P[A] = {}^4C_3 \times \dfrac{8}{12} \times \dfrac{8}{12} \times \dfrac{8}{12} \times \dfrac{4}{12}$

$P[A \cap B] = {}^3C_2 \times \dfrac{8}{12} \times \dfrac{8}{12} \times \dfrac{8}{12} \times \dfrac{4}{12}$

$\therefore \quad P[B / A] = \dfrac{P[A \cap B]}{P[A]} = \dfrac{{}^3C_2}{{}^4C_3} = \dfrac{3}{4}$

In case of sampling, without replacement

$P[A] = {}^4C_3 \times \dfrac{8}{12} \times \dfrac{7}{11} \times \dfrac{6}{10} \times \dfrac{4}{9}$

$P[A \cap B] = {}^3C_2 \times \dfrac{8}{12} \times \dfrac{7}{11} \times \dfrac{6}{10} \times \dfrac{4}{9}$

$\therefore \quad P[B / A] = \dfrac{{}^3C_2}{{}^4C_3} = \dfrac{3}{4}$

$\therefore$ The value of s is 4.

17. (1) : The inequality holds for $x = 4$. For $x \ge 5$

Now, ${}^{x-1}C_4 - {}^{x-1}C_3 < \dfrac{5}{4}({}^{x-2}P_2)$

$\Rightarrow \quad \dfrac{{}^{x-1}C_4}{{}^{x-2}P_2} - \dfrac{{}^{x-1}C_3}{{}^{x-2}P_2} < \dfrac{5}{4}$

$\Rightarrow \quad \dfrac{(x-1)!}{4!(x-5)!}\dfrac{(x-4)!}{(x-2)!} - \dfrac{(x-1)!}{3!(x-4)!}\dfrac{(x-4)!}{(x-2)!} < \dfrac{5}{4}$

$\Rightarrow \quad \dfrac{(x-1)(x-4)}{4!} - \dfrac{(x-1)}{3!} < \dfrac{5}{4}$

$\Rightarrow \quad x^2 - 5x + 4 - (x - 1)(4) < 30$

$\Rightarrow \quad x^2 - 9x - 22 < 0$

$\Rightarrow \quad (x - 11)(x + 2) < 0 \Rightarrow x < 11 \quad [\because \; x + 2 > 0]$

Thus, $4 \le x \le 10$. Thus the largest $x = 10$ and required sum is 1.

18. (0) : $a_k = 2a_{k-1} - a_{k-2} \Rightarrow a_{k-2} + a_k = 2a_{k-1}$

$\therefore \quad a_1, a_2, \ldots\ldots\ldots, a_{11}$ form an A.P.

$a_r = a_1 + (r - 1)d = 15 + (r - 1)d$

Now, $\dfrac{1}{11}\displaystyle\sum_{r=1}^{11} a_r^{\,2}$

$= \dfrac{1}{11}\displaystyle\sum_{r=1}^{11}(225 + (r-1)^2d^2 + 30(r-1)d)$

$= 225 + \dfrac{10 \cdot 21}{6}d^2 + \dfrac{30 \cdot 10d}{2} = 90$

$\Rightarrow \quad 35d^2 + 150d + 135 = 0 \Rightarrow 7d^2 + 30d + 27 = 0$

Solving, we get

$d = -3, \dfrac{-9}{7}$ and $a_2 = 15 + d = 15 - \dfrac{9}{7} = \dfrac{96}{7}$

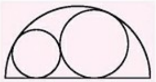
$\Rightarrow \quad 2a_2 = \dfrac{192}{7} \nless 27$

$\therefore \quad d = -3 \Rightarrow \dfrac{1}{11}(a_1 + a_2 + \ldots + a_{11})$

$= \dfrac{1}{2}(2a_1 + 10d) = \dfrac{1}{2}(30 - 30) = 0$

◆◆

OLYMPIAD CORNER



1. Two externally tangent circles of radii R_1 and R_2 are internally tangent to a semicircle of radius 1, as in the figure. Prove that $R_1 + R_2 \le 2(\sqrt{2}-1)$.

2. Ten players participate in a ping-pong tournament, in which every two players play against each other exactly once. If player i beats player j, player j beats player k, and player k beats player i, then the set $\{i, j, k\}$ is called a triangle. Let w_i and l_i denote the number of games won and lost, respectively, by the i^{th} player. Suppose that, whenever i beats j, then $l_i + w_j \ge 8$. Prove that there are exactly 40 triangles in the tournament.

3. In an equilateral triangle ABC (of side length 2) consider the incircle T.

   (a) Show that for all points P of T,
   $$\overline{PA}^2 + \overline{PB}^2 + \overline{PC}^2 = 5.$$

   (b) Show that for all points P of T, it is possible to construct a triangle of sides PA, PB, PC, with area $\sqrt{3}/4$.

4. Are there any nonconstant differentiable functions $f: R \to R$ such that $f(f(f(x))) = f(x) \ge 0$, for all $x \in R$?

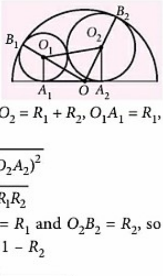
5. **(a)** Find all positive integers $p \le q \le r$ satisfying the equation $p + q + r + pq + qr + rp = pqr + 1$.

   (b) For each such solution (p, q, r), evaluate $\tan^{-1}(1/p) + \tan^{-1}(1/q) + \tan^{-1}(1/r)$.

6. Let n be a fixed positive integer. Show that for any nonnegative integer k, the equation
   $$x_1^3 + x_2^3 + \ldots + x_n^3 = y^{3k+2}$$
   has infinitely many solutions in positive integers x_i and y.

7. Suppose that u is a real parameter with $0 < u < 1$. Define
   $$f(x) = \begin{cases} 0 & \text{, if } 0 \le x \le u \\ 1-(\sqrt{ux}+\sqrt{(1-u)(1-x)})^2 & \text{, if } 0 \le x \le 1 \end{cases}$$
   and define the sequence $\{u_n\}$ recursively as follows: $u_1 = f(1)$, and $u_n = f(u_{n-1})$ for all $n > 1$.
   Show that there exists a positive integer k for which $u_k = 0$.

SOLUTIONS



1. Let O_1, O_2, and O denote the centres of the circles, and let A_1, A_2, B_1 and B_2 denote the points of tangency of these circles with the semicircle, as shown in the diagram. Then $O_1O_2 = R_1 + R_2$, $O_1A_1 = R_1$, and $O_2A_2 = R_2$.

So, $A_1A_2 = \sqrt{(O_1O_2)^2 - (O_1A_1 - O_2A_2)^2}$

$= \sqrt{(R_1+R_2)^2 - (R_1-R_2)^2} = 2\sqrt{R_1R_2}$

Also, $OB_1 = 1$, $OB_2 = 1$, $O_1B_1 = R_1$ and $O_2B_2 = R_2$, so that $OO_1 = 1 - R_1$ and $OO_2 = 1 - R_2$

Therefore, $A_1A_2 = OA_1 + OA_2$

$= \sqrt{(OO_1)^2 - (O_1A_1)^2} + \sqrt{(OO_2)^2 - (O_2A_2)^2}$

$= \sqrt{(1-R_1)^2 - R_1^2} + \sqrt{(1-R_2)^2 - R_2^2} = \sqrt{1-2R_1} + \sqrt{1-2R_2}$

Thus, $\sqrt{1-2R_1} + \sqrt{1-2R_2} = 2\sqrt{R_1R_2}$

Squaring, and then dividing each term by 2 and rearranging the terms, we get

$$\sqrt{(1-2R_1)(1-2R_2)} = 2R_1R_2 + R_1 + R_2 - 1$$

Squaring both sides and then simplifying, we get

$$8R_1R_2 = (2R_1R_2 + R_1 + R_2)^2 \qquad \text{...(i)}$$

So, $2\sqrt{2R_1R_2} = 2R_1R_2 + R_1 + R_2$ ...(ii)

Thus, $R_1+R_2=2\sqrt{R_1R_2}(\sqrt{2}-\sqrt{R_1R_2})$

$\le(R_1+R_2)(\sqrt{2}-\sqrt{R_1R_2})$, and therefore $\sqrt{R_1R_2}\le\sqrt{2}-1$.

Now, consider the function $f(x) = 2x(\sqrt{2}-x)$. $f(x)$ is increasing on the interval $(0,1/\sqrt{2})$ since $f'(x) = 2\sqrt{2}-4x > 0$ for x in the interval. Since $0<\sqrt{R_1R_2}\le\sqrt{2}-1<1/\sqrt{2}$ and $R_1+R_2=f(\sqrt{R_1R_2})$ from (ii), R_1+R_2 attains its maximum when $\sqrt{R_1R_2}=\sqrt{2}-1$.

Hence, $R_1+R_2\le 2(\sqrt{2}-1)[\sqrt{2}-(\sqrt{2}-1)]=2(\sqrt{2}-1)$.

2. Let P_i denote the i^{th} player, where $i = 1, 2,\ldots, 10$. Without loss of generality, assume that $w_1 \le w_2 \le \ldots \le w_{10}$.

Let $M = \max_{1\le i\le 10} w_i = w_{10}$ and $m = \min_{1\le i\le 10} w_i = w_1$. Since $w_i + l_i = 9$ for all i, the condition $l_i + w_j \ge 8$, whenever P_i beats P_j, is equivalent to

$$w_i \le w_j + 1. \qquad \text{...(i)}$$

$$\text{Clearly } \sum_{i=1}^{10} w_i = \binom{10}{2} = 45 \qquad \text{...(ii)}$$

We first show that

$(w_1, w_2, \ldots, w_{10}) = (4, 4, 4, 4, 4, 5, 5, 5, 5, 5)$

Since P_1 beats m players and loses to $9 - m$ others, we get from (i) and (ii) that $45 \le m + Mm + (9 - m)(m + 1)$ or $36 \le (M - m + 9)m$. ...(iii)

Similarly, since P_{10} beats M players and loses to $9 - M$ others, we have $45 \ge M + M(M - 1) + (9 - M)m = M^2 + (9 - M)m$. ...(iv)

Since $M \le 9$, from (iii) we obtain $36 \le (18 - m)m$, which implies $m \ge 3$. From (iii) we get $M^2 \le 45$, *i.e.*, $M \le 6$. If $M = 6$, then from (iv), $3m \le 9$ or $m \le 3$ and so $m = 3$. Similarly, if $m = 3$ then from (iii) $M \ge 6$ and so $M = 6$. Since 10×45, $M \ne m$. Therefore, the only possibilities are $(M, m) = (6, 3)$ and $(5, 4)$.

Note : At this point, Lau listed all 10-tuples $(w_1, w_2, \ldots, w_{10})$ satisfying $(M, m) = (6, 3)$ and (2), and then showed that none were possible. But the case $(M, m) = (6, 3)$ can be eliminated as follows. If $m = w_1 = 3$, then six players beat P_1, so by (i) at least seven of the w_i's (including w_1) are ≤ 4. If $M = w_{10} = 6$, then P_{10} beats six players, so by (i) atleast seven of the w_i's (including w_{10}) are ≥ 5. But since there are only ten w_i's this is impossible.

Thus $(M, m) = (5, 4)$, and $(4, 4, 4, 4, 4, 5, 5, 5, 5, 5)$ is the only possible solution. By exhibiting the tournament graphically, one can see that this is indeed a solution. Now, there is a theorem in graph theory which states that the number of transitive triples [3 players P_i, P_j and P_k such that P_i beats P_j, P_j beats P_k, and P_i beats P_k – *Ed.*] in a tournament with n vertices and score sequence $(w_1, w_2, \ldots, w_n)$ is $\sum_{i=1}^{n} w_i(w_i - 1)/2$.

Note : This result is true because the number of such triples is just the number of unordered pairs $\{P_j, P_k\}$ of (distinct) players beaten by P_i, which is $\binom{w_i}{2}$, added up over all P_i.] It follows that, with $n = 10$, the number of triangles (cyclic triples) is

$$\binom{10}{3}-\frac{1}{2}\sum_{i=1}^{n} w_i(w_i-1)=120-\frac{1}{2}(5)(5)(5-1)-\frac{1}{2}(5)(4)(4-1)$$

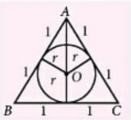
$= 40.$

This completes the proof.

3. Take O as origin, with the x-axis parallel to BC, and y-axis along OA.

Then the incircle has radius $r = 1/\sqrt{3}$ and A, B, C have coordinates

$$A\left(0,\frac{2}{\sqrt{3}}\right), B\left(-1,\frac{-1}{\sqrt{3}}\right), C\left(1,\frac{-1}{\sqrt{3}}\right).$$



A point on the incircle has coordinates parameterized by θ, $0 \le \theta < 2\pi$ given by $P\left(\frac{1}{\sqrt{3}}\cos\theta, \frac{1}{\sqrt{3}}\sin\theta\right)$.

(a) Let $c = \cos\theta$, $s = \sin\theta$. Then

$AP^2 + BP^2 + CP^2$

$$=\left[\frac{1}{3}c^2+\frac{1}{3}(2-s)^2\right]+\left[\frac{1}{3}(c+\sqrt{3})^2+\frac{1}{3}(s+1)^2\right]$$
$$\left[\frac{1}{3}(c-\sqrt{3})^2+\frac{1}{3}(s+1)^2\right]$$

$$=\frac{1}{3}[c^2+4-4s^2+s^2+c^2+2\sqrt{3}c+3+$$
$$s^2+2s+1+c^2-2\sqrt{3}c+3+s^2+2s+1]$$

$$=\frac{1}{3}[3(c^2+s^2)+12]=5$$

since $c^2 + s^2 = 1$.

(b) Now, set $x = AP = \frac{1}{\sqrt{3}}\sqrt{5-4s}$,

$y = BP = \frac{1}{\sqrt{3}}\sqrt{5+2s+2\sqrt{3}c}$, and

$z = CP = \frac{1}{\sqrt{3}}\sqrt{5+2s-2\sqrt{3}c}.$

By reflection and rotational geometry, the distances AP, BP, CP will be a permutation of those obtained when $\pi/6 \le \theta \le \pi/2$, so that $1/\sqrt{3} \le x \le 1$.

Also,

$$y=\frac{1}{\sqrt{3}}\sqrt{5+4\left(\frac{1}{2}s+\frac{\sqrt{3}}{2}c\right)}=\frac{1}{\sqrt{3}}\sqrt{5+4\cos\left(\theta-\frac{\pi}{6}\right)}$$

so that $\sqrt{7}/\sqrt{3} \le y \le \sqrt{3}$ and

$$z=\frac{1}{\sqrt{3}}\sqrt{5+4\left(\frac{1}{2}s-\frac{\sqrt{3}}{2}c\right)}=\frac{1}{\sqrt{3}}\sqrt{5+4\sin\left(\theta-\frac{\pi}{3}\right)}$$

so that $\sqrt{5}/\sqrt{3} \le z \le \sqrt{7}/\sqrt{3}$.

Thus, $(x+y)_{\min}=\dfrac{1+\sqrt{7}}{\sqrt{3}}>z_{\max}=\dfrac{\sqrt{7}}{\sqrt{3}}$

and $(x+z)_{\min}=\dfrac{\sqrt{7}+\sqrt{5}}{\sqrt{3}}>x_{\max}=1$

and $(z+x)_{\min}=\dfrac{1+\sqrt{5}}{\sqrt{3}}>y_{\max}=\sqrt{3}$,

and x, y, z (*i.e.* AP, BP, CP) can form the sides of a triangle.

From Heron's formula the area, F, of this triangle is given by

$$F^2=\frac{1}{2}(x+y+z)\frac{1}{2}(-x+y+z)\cdot\frac{1}{2}(x-y+z)\cdot\frac{1}{2}(x+y-z)$$

$$=\frac{1}{16}[(y+z)^2-x^2][x^2-(y-z)^2]$$

$$=\frac{1}{16}[\frac{1}{3}(5+2s+2\sqrt{3}c+5+2s-2\sqrt{3c}-5+4s)$$

$$+2\cdot\frac{1}{3}(5+2s+2\sqrt{3}c)^{1/2}(c+2s-2\sqrt{3}c)^{1/2}]$$

$$\cdot[\frac{1}{3}(5-4s-5-2s-2\sqrt{3}c-5-2s+2\sqrt{3}c)$$

$$+2\cdot\frac{1}{3}(5+2s+2\sqrt{3}c)^{1/2}(5+2s-2\sqrt{3}c)^{1/2}]$$

$$=\frac{1}{48}\left[(5+8s)+2\sqrt{(5+2s)^2-12c^2}\right]$$

$$\left[-(5+8s)+2\sqrt{(5+2s)^2-12c^2}\right]$$

$$=\frac{1}{48}[100+80s+16s^2-48c^2-25-80s-64s^2]$$

$$=\frac{1}{48}[75-48]=\frac{9}{16}\text{, since } c^2+s^2=1.\text{ Thus } F=3/4.$$

4. Applying f to both sides of the functional equation

$$f(f(f(x)))=f(x)\ge 0 \qquad \text{...(i)}$$

gives $g(g(x))=g(x)$, where $g(x)=f(f(x))$ for all $x\in R$. Of course g is also a differentiable function on R and $g(x)\ge 0$ for all $x\in R$. Then the range $T=g(R)$ of g is an interval contained in $[0,+\infty)$. Let a be the infimum of T. Since $g(t)=t$ for all $t\in T$ and g is continuous, it follows that $g(a)=a$. Assuming that T has more than one element, choose $\delta>0$ such that $(a, a+\delta)\subseteq T$. Then $x\in(a-\delta, a)$ implies $g(x)\ge g(a)$ $(=a)$, hence

Therefore, $\dfrac{g(x)-g(a)}{x-a}\le 0$.

$$g_L'(a)=\lim_{x\to a^-}\frac{g(x)-g(a)}{x-a}\le 0. \qquad \text{...(ii)}$$

For $x\in(a, a+\delta)$ we have

$$\frac{g(x)-g(a)}{x-a}=1.$$

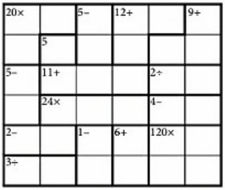
Hence, $g_R'(a)=\lim\limits_{x\to a^+}\dfrac{g(x)-g(a)}{x-a}=1.$ ...(iii)

PUZZLE CORNER

MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics. In this puzzle 6 × 6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is combined using a specified operation on its numbers. For example, if that value is 3 for a two-box cluster and operation is multiply, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.



20×		5–	12+		9÷
	5				
5–	11+			2÷	
	24×			4–	
2–		1–	6+	120×	
3÷					

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

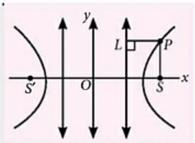
CONCEPT MAP

HYPERBOLA

Class XI

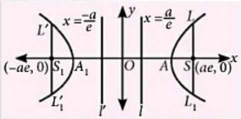
Definition

- Locus of a point which moves in such a way that the difference of its distance from two fixed points (foci) is always constant.
- General equation of second degree $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a hyperbola if $\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} \neq 0$ and $h^2 > ab$.



Standard form

Standard form is given by $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$



- **Centre :** $O(0, 0)$
- **Foci :** $S(ae, 0), S_1(-ae, 0)$
- **Vertices :** $A(a, 0), A_1(-a, 0)$
- **Directrices :** $x = a/e$ and $x = -a/e$
- **Transverse axis :** $AA_1 = 2a$; **Conjugate axis** $= 2b$
- **Eccentricity :** $e = \sqrt{\frac{a^2 + b^2}{a^2}}$
- **Latus rectum :** $LL_1 = L'L'_1 = \frac{2b^2}{a} = 2a(e^2 - 1)$

Rectangular Hyperbola

A hyperbola is said to be rectangular hyperbola, if the asymptotes are at right angle *i.e.*, $\theta = 90°$. Equation of rectangular hyperbola is $x^2 - y^2 = a^2$ or $xy = c^2$.

Conjugate Hyperbola

If $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is a hyperbola, then its conjugate hyperbola is given by $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$

Properties

If $A\left(ct_1, \frac{c}{t_1}\right)$ and $B\left(ct_2, \frac{c}{t_2}\right)$ are the points lying on the hyperbola $xy = c^2$, then

- Vertices of $xy = c^2$ are $(c, c), (-c, -c)$
- Foci : $(\pm c\sqrt{2}, \pm c\sqrt{2})$
- Equation of directrices is $x + y \pm c\sqrt{2} = 0$
- Slope of chord at $A = -\frac{1}{t_1}$
- Chord subtend right angle at the vertices.
- Slope of tangent at $A = -\frac{1}{t_1^2}$
- Locus of the point of intersection of tangents at the end point of chords is $x + y = 0$.

Important Results

Let $S \equiv \frac{x^2}{a^2} - \frac{y^2}{b^2} - 1, S_1 \equiv \frac{x_1 x}{a^2} - \frac{y_1 y}{b^2} - 1,$

$S_{11} \equiv \frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} - 1, S_2 \equiv \frac{x_2 x}{a^2} - \frac{y_2 y}{b^2} - 1, S_{22} \equiv \frac{x_2^2}{a^2} - \frac{y_2^2}{b^2} - 1, S_{12} \equiv \frac{x_1 x_2}{a^2} - \frac{y_1 y_2}{b^2} - 1,$

- Location of $P(x_1, y_1)$: P is inside the hyperbola if $S_{11} > 0$.
  P is outside hyperbola if $S_{11} < 0$ and P is on the hyperbola if $S_{11} = 0$.
- Chord with midpoint $P(x_1, y_1) : S_1 = S_{11}$
- Chord joining the points $P(x_1, y_1)$ and $Q(x_2, y_2)$: $S_1 + S_2 = S_{12}$
- Tangent at $P(x_1, y_1) : S_1 = 0$
- Chord of contact of tangents from $P(x_1, y_1) : S_1 = 0$
- Tangents with slope m are $y = mx \pm \sqrt{a^2 m^2 - b^2}$.
- Pair of tangents from $P(x_1, y_1) : S_1^2 = S_{11} S$
- Normal at $P(x_1, y_1)$: $\frac{a^2 x}{x_1} + \frac{b^2 y}{y_1} = a^2 + b^2 = a^2 e^2$

PROBABILITY

Class XII

CONCEPT MAP

PROBABILITY

Theorems

Multiplication

- $P(A \cap B) = P(A) \cdot P(B \mid A) = P(B) \cdot P(A \mid B)$, provided $P(A) \neq 0$, $P(B) \neq 0$ *i.e.*, $P(AB) = P(A) \cdot P(B \mid A) = P(B) \cdot P(A \mid B)$, provided $P(A) \neq 0$, $P(B) \neq 0$.

Total Probability

- $P(A) = P(E_1)\, P(A \mid E_1) + P(E_2)\, P(A \mid E_2) + \ldots\ldots + P(E_n)\, P(A \mid E_n)$ where, $E_1, E_2, E_3, \ldots\ldots, E_n$ are mutually exclusive and exhaustive events.

Bayes'

- $P(E_i|A) = \dfrac{P(E_i)\, P(A|E_i)}{\Sigma P(E_i)\, P(A|E_i)}$, where $i = 1, 2, 3, \ldots\ldots, n$, where, $E_1, E_2, E_3, \ldots\ldots, E_n$ are mutually exclusive and exhaustive events.

Independent Events

Occurrence or non-occurrence of one does not affect the occurrence of the other *i.e.*,

$P(A \mid B) = P(A)$ provided $P(B) \neq 0$,

$P(B \mid A) = P(B)$ provided $P(A) \neq 0$

[For n events, $P(A_1 \cap A_2 \cap \ldots\ldots \cap A_n) = P(A_1) \cdot P(A_2) \ldots\ldots P(A_n)$]

Random Variables and its Probability Distributions

- A real valued function, whose domain is the sample space of a random experiment. Generally, it is denoted by X.
- Let real numbers $x_1, x_2, \ldots, x_n$ are the possible values of a random variable X and $p_1, p_2, \ldots, p_n$ are the corresponding probabilities to each value of the random variable X. Then the probability distribution is

$$\begin{array}{lcccc} X: & x_1 & x_2 & \ldots & x_n \\ P(X): & p_1 & p_2 & \ldots & p_n. \end{array}$$

- $P(x_k)$ *i.e.*, $P(X = x_k)$ lies between 0 and 1 for $k = 1, 2, \ldots, n$.
- $\sum_{i=1}^{n} p_i = 1$

Mean

- $\bar{X} = E(X) = \mu = p_1x_1 + p_2x_2 + \ldots + p_nx_n = \sum_{i=1}^{n} p_ix_i$

Conditional Probability

- Probability of occurrence of an event A, given that B has already occurred. *i.e.*, $P(A|B) = P(A \cap B) / P(B)$

Properties

- $0 \leq P(A|B) \leq 1$
- $P((A \cup B)|F) = P(A|F) + P(B|F) - P((A \cap B)|F)$, where $P(F) \neq 0$
  $= P(A|F) + P(B|F)$ (if A & B are disjoint)
- $P(A'|B) = 1 - P(A|B)$

Variance & Standard Deviation

- $\text{Var}\,(X) = \sigma^2 = \sum_{i=1}^{n} p_ix_i^2 - \left(\sum_{i=1}^{n} p_ix_i\right)^2$
  or $\text{Var}(X) = E(X^2) - \{E(X)\}^2$
- Standard deviation $(\sigma) = \sqrt{\text{Var}(X)}$

Bernoulli's Trials

Trials of a random experiment are called bernoulli's trials, if

- they are finite in number.
- they are independent of each other.
- each trial has exactly two outcomes *i.e.*, success or failure.
- the probability of success or failure remains same in each trial.

Binomial Distribution

$P(X = r) = {}^nC_r\, p^r q^{n-r}$, where $r = 0, 1, 2, \ldots, n$, where p and q are probabilities of success and failure respectively.

Also, $p + q = 1$

Mean

- $\bar{X} = E(x) = np$

Variance

- $\sigma^2 = npq$

Standard Deviation

- $\sigma = \sqrt{npq}$

- Mean is always greater than variance.

(ii) and (iii) are contradictory since g is differentiable at a. We are led to the conclusion that T is a single point, *i.e.*, g is a constant function, say $g(x) = c$ for all $x \in R$. This gives, using (i), that $f(c) = f(x)$ for all $x \in R$, showing that f is a constant function. Thus there is no nonconstant differentiable function satisfying (i).

5. **(a)** There are exactly 3 solutions, given by

$(p, q, r) = (2, 4, 13), (2, 5, 8)$ and $(3, 3, 7)$.

Note first that

$$(p - 1)(q - 1)(r - 1) = pqr - (pq + qr + rp) + (p + q + r) - 1$$

which becomes, using the given equation,

$$(p - 1)(q - 1)(r - 1) = 2(p + q + r - 1). \quad \text{...(i)}$$

If $p \geq 4$, then $4 \leq p \leq q \leq r$ implies

$(p - 1)(q - 1)(r - 1) \geq 9(r - 1)$ and

$$2(p + q + r - 1) \leq 2(3r - 1);$$

and since, $9(r - 1) - 2(3r - 1) = 3r - 7 > 0$,

(i) cannot hold in this case. Thus $p < 4$. Since $p = 1$ clearly does not satisfy (i), we have $p = 2$ or 3. When $p = 2$, (i) becomes $(q - 1)(r - 1) = 2(q + r + 1)$ or $(q - 3)(r - 3) = 10$. Thus $q - 3 = 1, r - 3 = 10$ or $q - 3 = 2$, $r - 3 = 5$, yielding two solutions: $(2, 4, 13)$ and $(2, 5, 8)$. When $p = 3$, (i) becomes $(q - 1)(r - 1) = q + r + 2$ or $(q - 2)(r - 2) = 5$ which yields the third solution: $(3, 3, 7)$.

(b) The value is $\pi/4$ in all cases. To see this, set $A = \tan^{-1}(1/p)$, $B = \tan^{-1}(1/q)$ and $C = \tan^{-1}(1/r)$

Since $0 < \frac{1}{r} \leq \frac{1}{q} \leq \frac{1}{p} < 1$,

We have, $0 < C \leq B \leq A < \pi/4$

$\therefore \; 0 < A + B + C < 3\pi/4$.

From the well known formula for $\tan(x + y)$ one easily deduces that for all x, y, z with $x + y + z \neq k\pi + \pi/2$ (where k denotes an integer),

$$\tan(x+y+z) = \frac{\tan x + \tan y + \tan z - \tan x \tan y \tan z}{1 - (\tan x \tan y + \tan y \tan z + \tan z \tan x)}$$

$\therefore \; \tan(A + B + C)$

$$= \frac{\frac{1}{p} + \frac{1}{q} + \frac{1}{r} - \frac{1}{pqr}}{1 - \left(\frac{1}{pq} + \frac{1}{qr} + \frac{1}{rp}\right)} = \frac{pq + qr + rp - 1}{pqr - (p + q + r)} = 1.$$

Hence, $A + B + C = \pi/4$

$$\therefore \; \tan^{-1}(1/p) + \tan^{-1}(1/q) + \tan^{-1}(1/r) = \frac{\pi}{4}$$

6. Since, $1^3 + 2^3 + \ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$

we see that when $k = 0$,

$$(x_1, x_2, \ldots, x_n; y) = \left(1, 2, \ldots, n; \frac{n(n+1)}{2}\right)$$

is a solution. To see that we can generate infinitely many solutions. In general, set $c = (n(n + 1))/2$ and notice that for all positive integers q, we have:

$$(c^k q^{3k+2})^3 + (2c^k q^{3k+2})^3 + \ldots + (nc^k q^{3k+2})^3$$
$$= c^{3k} q^{3(3k+2)} (1^3 + 2^3 + \ldots + n^3)$$
$$= c^{3k} q^{3(3k+2)} \left(\frac{n(n+1)}{2}\right)^2 = c^{3k+2} q^{3(3k+2)} = (cq^3)^{3k+2}.$$

That is, $(x_1, x_2, \ldots, x_n; y) = (c^k q^{3k+2}, 2c^k q^{3k+2}, \ldots, nc^k q^{3k+2}; cq^3)$ is a solution. This completes the proof.

7. Note first that $u_1 = 1 - u$. Since for all $x \in [u, 1]$, $u \leq x$ and $1 - x \leq 1 - u$ we have

$$1 - (\sqrt{ux} + \sqrt{(1-u)(1-x)})^2$$
$$= 1 - ux - (1-u)(1-x) - 2\sqrt{ux(1-u)(1-x)}$$
$$= u + x - 2ux - 2\sqrt{ux(1-u)(1-x)}$$
$$\leq u + x - 2ux - 2u(1 - x) = x - u.$$

Therefore, $f(x) = 0$ if $0 \leq x \leq u$ ...(i)

and $f(x) \leq x - u$ if $u \leq x \leq 1$. ...(ii)

From (ii) we get $u_2 = f(u_1) - u = 1 - 2u$ if $u_1 \geq u$. An easy induction then yields $u_{n+1} = f(u_n) \leq u_n - u \leq 1 - (n + 1)u$ if $u_i \geq u$ for all $i = 1, 2, \ldots, n$.

Thus for sufficiently large k, we must have $u_{k-1} < u$ and then $u_k = f(u_{k-1}) = 0$ by (ii).

◆◆

PUZZLE CORNER

ANSWER · MARCH 2021

2÷ 4	5+ 2	3	36× 1	6	5 5
2	12× 3	4	6	4– 5	1
6+ 1	5	2÷ 2	4	3 3	4– 6
3÷ 3	1	6 6	1– 5	4	2
30× 6	1– 4	5	4× 2	1	12× 3
5	5– 6	1	3 3	2	4

Exam on 3rd July

JEE PRACTICE QUESTIONS ADVANCED 2021

*ALOK KUMAR

SINGLE OPTION CORRECT TYPE

1. The function $y = x - \cot^{-1} x - \log\left(x - \sqrt{x^2+1}\right)$ is increasing on

(a) (0, 0) (b) $(-\infty, 0)$ (c) $(0, \infty)$ (d) $(-\infty, \infty)$

2. An infinite G. P. has 2nd term x and its sum is 4, then x belongs to

(a) (0, 2] (b) (1, 8)
(c) (–8, 1] (d) none of these

3. $\sum_{r=0}^{10} 2^r\, {}^{20}C_r\ {}^{20-r}C_{10-r}$ is equal to

(a) ${}^{20}C_{10}$ (b) ${}^{20}C_{10}\left(\frac{3}{2}\right)^{10}$
(c) ${}^{20}C_{10}\ 3^{10}$ (d) ${}^{20}C_{10}\ 2^{10}$

4. If the graphs of the functions $y = \ln x$ and $y = ax$ intersect at exactly two points, then a must be

(a) $(0, e)$ (b) $(1/e, 0)$
(c) $(0, 1/e)$ (d) none of these

5. If $\int_0^{\pi/2} \frac{dx}{a^2\cos^2 x + b^2 \sin^2 x} = \frac{\pi}{2ab}$, then the value of $\int_0^{\pi/2} \frac{dx}{(4\cos^2 x + 9\sin^2 x)^2}$ is equal to

(a) $\frac{11\pi}{864}$ (b) $\frac{13\pi}{864}$
(c) $\frac{17\pi}{864}$ (d) none of these

6. If $(3 + x^{2008} + x^{2009})^{2010} = a_0 + a_1 x + a_2 x^2 + ... + a_n x^n$, then the value of $a_0 - \frac{1}{2}a_1 - \frac{1}{2}a_2 + a_3 - \frac{1}{2}a_4 - \frac{1}{2}a_5 + a_6 - ...$ is

(a) 3^{2010} (b) 1
(c) 2^{2010} (d) none of these

7. Let AB be any chord of the circle $x^2 + y^2 - 4x - 4y + 4 = 0$ which subtends an angle of 90° at the point (2, 3), then the locus of the mid point of AB is a circle whose centre is

(a) (1, 5) (b) (1, 5/2)
(c) (1, 3/2) (d) (2, 5/2)

8. If k be a positive integer, then the number of values of k satisfying

$$\int_0^{\pi/2}\left[k^2\left(\cos 3x + \frac{1}{2}\cos x\right) + \sin x - 2k\cos x\right]dx \le 1, \text{ is}$$

(a) 10 (b) 11
(c) 12 (d) none of these

9. The solution of differential equation
$(x\cot y + \ln\cos x)dy + (\ln\sin y - y\tan x)dx = 0$ is

(a) $(\sin x)^y (\cos y)^x = c$ (b) $(\sin y)^x (\cos x)^y = c$
(c) $(\sin x)^y (\sin y)^x = c$ (d) $(\cot x)^y (\cot y)^x = c$

10. The value of $\sum_{r=0}^{n-1}\left(\frac{n+1}{n}\right)\left(\frac{r.\,{}^nC_r\ {}^nC_{r+1}}{r+2}\right)$ is

(a) ${}^{2n-1}C_{n+1}$ (b) ${}^{2n}C_{n-1}$
(c) ${}^{2n-1}C_{n-1}$ (d) ${}^{2n}C_{n-2}$

11. The number of points in the cartesian plane with integral co-ordinates satisfying the inequations $|x| \le 10$, $|y| \le 10$ and $|x - y| \le 10$ is

(a) 321 (b) 331
(c) 341 (d) none of these

12. If $f(x + y) = f(x) - f(y) + 2xy - 1\ \forall\ x, y \in R$. Also if $f(x)$ is differentiable and $f'(0) = b$ also $f(x) > 0\ \forall\ x$, then the set of values of b is

(a) ϕ (b) {1}
(c) {1, 2} (d) none of these

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*Alok Kumar, a B.Tech from IIT Kanpur and INMO 4th ranker of his time, has been training IIT and Olympiad aspirants for close to two decades now. His students have bagged AIR 1 in IIT JEE and also won medals for the country at IMO. He has also taught at Maths Olympiad programme at Cornell University, USA and UT, Dallas. He has been regularly proposing problems in international Mathematics journals.

13. Let $a_1 = \left(\tan\frac{\pi}{8}\right)^{\tan\frac{\pi}{8}}$, $a_2 = \left(\tan\frac{\pi}{8}\right)^{\cot\frac{\pi}{8}}$, $a_3 = \left(\cot\frac{\pi}{8}\right)^{\cot\frac{\pi}{8}}$, $a_4 = \left(\cot\frac{\pi}{8}\right)^{\tan\frac{\pi}{8}}$, then which of the following statement(s) is/are true?

(a) $a_4 > a_3 > a_2 > a_1$ (b) $a_3 > a_4 > a_2 > a_1$

(c) $a_3 > a_4 > a_1 > a_2$ (d) $a_3 > a_1 > a_2 > a_4$

14. The equation of curve passing through (1, 1) in which the subtangent is always bisected at the origin, is

(a) $y^2 = x$ (b) $2x^2 - y^2 = 1$

(c) $x^2 + y^2 = 2$ (d) $x + y = 2$

15. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors such that $\vec{b} \times \vec{c} = \vec{a}$, $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{c} \times \vec{a} = \vec{b}$ then

(a) $[\vec{a}\ \vec{b}\ \vec{c}] = 1$ (b) $[\vec{a}\ \vec{b}\ \vec{c}] \neq 1$

(c) $|\vec{a}| + |\vec{b}| + |\vec{c}| = 5$ (d) none of these

16. If in a ΔABC, $\cos A + 2\cos B + \cos C = 2$, then a, b, c are in

(a) A.P. (b) H.P. (c) G.P. (d) A.G.P.

17. The value of $a (a < 0)$ for which least value of quadratic expression $4x^2 - 4ax + a^2 - 2a + 2$ on the interval $0 \le x \le 2$ is equal to 3, is

(a) $-\sqrt{2}$ (b) $-\sqrt{2} - 2$

(c) $1 - \sqrt{2}$ (d) none of these

18. If $f(x) = \tan\frac{3\pi}{2}[x] + \cos^2 \pi[x] + e^{\left\{\frac{x}{3}\right\}}$ (where [.] denotes the greatest integer function and {.} denotes fractional part), then

(a) $f(x)$ is periodic with fundamental period not defined

(b) $f(x)$ is periodic with period 6

(c) $f(x)$ is non periodic

(d) $f(x)$ is periodic with period 2

19. If the shortest distance between lines $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda_1(2\hat{i} + 3\hat{j} + 4\hat{k})$ and $\vec{r} = 2\hat{i} + 4\hat{j} + 5\hat{k} + \lambda_2(3\hat{i} + 4\hat{j} + 5\hat{k})$ is x.

Then $\cos^{-1}\cos\sqrt{6}\,x$ is equal to

(a) 1/2 (b) 0 (c) 1 (d) π

20. The maximum values of $\frac{\log x}{x}$ is

(a) 1 (b) $2/e$ (c) e (d) $1/e$

21. The maximum value of $\cos(\sin x) + \sin(\cos x)$, $0 \le x \le \pi$, is

(a) $1 + \sin 1$ (b) $\cos 1$

(c) $1 + \cos 1$ (d) $2\sin^2\frac{1}{2}$

22. The sides of the triangle are in the ratio $1 : \sqrt{3} : 2$, then the corresponding angles of triangle are in the ratio

(a) 1 : 3 : 4 (b) 2 : 3 : 4

(c) 3 : 2 : 1 (d) 1 : 2 : 3

ONE OR MORE THAN ONE OPTION(S) CORRECT TYPE

23. *ABCD* is a cyclic quadrilateral, then the correct statements are

(a) if *B* lies inside the circle through *A*, *C* and *D*, then *D* lies inside the circle through *A*, *B* and *C*.

(b) if *B* lies outside the circle through *A*, *C* and *D*, then *D* lies outside the circle through *A*, *B* and *C*.

(c) if *B* lies inside the circle through *A*, *C* and *D*, then *D* lies outside the circle through *A*, *B* and *C*.

(d) if *B* lies outside the circle through *A*,*C* and *D*, then *D* lies inside the circle through *A*, *B* and *C*.

24. Triangles $A_1A_2A_3$ and $B_1B_2B_3$ have side lengths (a_1, a_2, a_3) and (b_1, b_2, b_3) respectively satisfying the relation

$$\sqrt{a_1 + a_2 + a_3}\sqrt{b_1 + b_2 + b_3} = \sqrt{a_1b_1} + \sqrt{a_2b_2} + \sqrt{a_3b_3},$$

then which one of the following statements is/are true ?

(a) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$ (b) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3} = 1$

(c) $\Delta A_1A_2A_3$ and $\Delta B_1B_2B_3$ are similar

(d) $\Delta A_1A_2A_3$ and $\Delta B_1B_2B_3$ are congruent

25. If the equations $\bar{a}z + a\bar{z} + b = 0$ and $\bar{a}z - a\bar{z} + b_1 = 0$ represent two lines L_1 and L_2 in the complex plane then

(a) L_1 and L_2 are perpendicular

(b) b is purely real

(c) b_1 is purely imaginary

(d) b_1 is purely real

26. Let $f : R \to R$ be a function defined by $f(x+1) = \frac{f(x) - 5}{f(x) - 3} \forall \in R$. Then which of the following statement(s) is/are true?

(a) $f(2008) = f(2004)$ (b) $f(2006) = f(2010)$

(c) $f(2006) = f(2002)$ (d) $f(2006) = f(2018)$

27. If the equation $\sec x + \operatorname{cosec} x = k$, $(k \in R)$ has exactly 3 solutions between 0 and π, then k can take values

(a) $\sqrt{2}$ (b) 2 (c) $3\sqrt{2}$ (d) 4

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COMPREHENSION TYPE

Let $\vec{r}$ is a position vector of a variable point in cartesian OXY plane such that $\vec{r}\cdot(10\hat{j}-8\hat{i}-\vec{r})=40$ and $p_1=\max\{|\vec{r}+2\hat{i}-3\hat{j}|^2\}$, $p_2=\min\{|\vec{r}+2\hat{i}-3\hat{j}|^2\}$.

A tangent line is drawn to the curve $y=\frac{8}{x^2}$ at the point A with abscissa 2. The drawn line cuts x-axis at a point B.

28. p_2 is equal to

(a) 9
(b) $2\sqrt{2}-1$
(c) $6\sqrt{2}+3$
(d) $9-4\sqrt{2}$

29. p_1+p_2 is equal to

(a) 2 (b) 10 (c) 18 (d) 5

30. $\overline{AB}\cdot\overline{OB}$ is

(a) 1 (b) 2 (c) 3 (d) 4

MATRIX-MATCH TYPE

31. Match the following :

Column-I		Column-II	
(A)	If $y=\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}(b)$, $(0 < b < 1)$ and $0 < y \le \pi/4$, then the maximum value of b will be	(1)	3
(B)	The number of solutions of $\sin^4 x+\cos^3 x \ge 1$ in $(0, 2\pi)$ will be	(2)	1/3
(C)	The area bounded by the loop of $4y^2 = x^2(4-x^2)$ is	(3)	1/2
(D)	If $(1+x^2)\frac{dy}{dx}+2xy-1=0$ passes through origin, then $y(1)$ will be	(4)	16/3

32. Match the following :

Column-I		Column-II	
(A)	The differential equation $y=x\frac{dy}{dx}+\frac{dx}{dy}$ represents	(1)	pair of straight lines
(B)	If an ellipse slides between two perpendicular straight lines, then the locus of its centre is	(2)	straight line
(C)	If a, b are real numbers and $c > 0$, then the locus of (x,y) represented by the equation $\lvert ay-bx\rvert=c\sqrt{(x-a)^2+(y-b)^2}$ is	(3)	circle
(D)	If $h \ne 0$ and $\frac{a}{h}=\frac{\sqrt{2}c}{b}$, then the equation $hxy + ax + by + c = 0$ represents	(4)	parabola
		(5)	hyperbola

33. Match the following :

Column-I		Column-II	
(A)	$\bar{a}$, $\bar{b}$ are unit vectors and $(\bar{a}+2\bar{b})\perp(5\bar{a}-4\bar{b})$, then $2(\bar{a}\cdot\bar{b})$ is equal to	(1)	0
(B)	The points (1, 0, 3), (–1, 3, 4), (1, 2, 1), $(k, 2, 5)$ are coplanar, then $k =$	(2)	–1
(C)	The vectors $(1, 1, m)$, $(1, 1, m+1)$, $(1, -1, m)$ are coplanar, then number of values of m is	(3)	1
(D)	$\vec{a}\times(\vec{b}\times\vec{c})+\vec{b}\times(\vec{c}\times\vec{a})+\vec{c}\times(\vec{a}\times\vec{b})$ is equal to	(4)	2

34. Match the following :

Column-I		Column-II	
(A)	If the line $\frac{x-1}{1}=\frac{y+1}{-2}=\frac{z+1}{\lambda}$ lies in the plane $3x-2y+5z=0$, then x is equal to	(1)	$\sin^{-1}\sqrt{\frac{6}{25}}$
(B)	If $(3, \lambda, \mu)$ is a point on the line $2x + y + z - 3 = 0 = x - 2y + z - 1$, then $\lambda + \mu$ is equal to	(2)	–7/5
(C)	The angle between the line $x = y = z$ and the plane $4x - 3y + 5z = 2$ is	(3)	–3
(D)	The angle between the planes $x + y + z = 0$ and $3x - 4y + 5z = 0$ is	(4)	$\cos^{-1}\sqrt{\frac{8}{75}}$

35. Match the following :

Column-I		Column-II	
(A)	The number of arrangements of the word BANANA in which two N's do not appear together is	(1)	[1, 2)
(B)	If $(1+x)^n=C_0+C_1x+\ldots+C_nx^n$, then $\left(1+\frac{C_1}{C_0}\right)\ldots\left(1+\frac{C_n}{C_{n-1}}\right)$ is	(2)	$\frac{(n+1)^n}{n!}$

(C)	If c, d, e are in A.P., then $\Delta=\begin{vmatrix} x+1 & x+2 & x+c \\ x+2 & x+3 & x+d \\ x+3 & x+4 & x+e \end{vmatrix}$ is	(3)	$\frac{2(n+1)^n}{n!}$
(D)	$\frac{c^2+d^2+e^2}{cd+de+ce}$ lies between (where c, d, e are the lengths of the sides of ΔCDE)	(4)	0
		(5)	40

NUMERICAL VALUE TYPE

36. If $f(x+y+z)=f(x)+f(y)+f(z)$ with $f(1)=1$ and $f(2)=2$ and $x, y, z \in N$, then $\lim_{n\to\infty}\frac{\sum_{r=1}^{n}(4r)\,f(3r)}{n^3}$ is equal to ________.

37. $x, y \in R$, $x^2+y^2+xy=1$, then the minimum value of x^3y+xy^3+4 is _____.

38. If the equation of the curve on reflection of the ellipse $\frac{(x-4)^2}{16}+\frac{(y-3)^2}{9}=1$ about the line $x-y-2=0$ is $16x^2+9y^2+k_1x-36y+k_2=0$, then $\frac{k_1+k_2}{44}$ is ______.

39. If the number of ordered pairs of (x, y) satisfying the system of equations $5x\left(1+\frac{1}{x^2+y^2}\right)=12$ and $5y\left(1-\frac{1}{x^2+y^2}\right)=4$ is n, then n is _______.

40. If x, y, z are distinct positive numbers such that $x+\frac{1}{y}=y+\frac{1}{z}=z+\frac{1}{x}$, then the value of xyz is ______.

SOLUTIONS

1. **(d) :** $y=x-\cot^{-1}x-\log\left(x-\sqrt{x^2+1}\right)$,

$\frac{dy}{dx}=1+\frac{1}{1+x^2}+\frac{1}{\sqrt{x^2+1}}$ as $\frac{dy}{dx}>0$ for all R,

So, y is increasing on $(-\infty, \infty)$.

2. **(c) :** Sum$=\frac{\frac{x}{r}}{1-r}=4$ (where r is common ratio)

$\Rightarrow x=4r(1-r)=4(r-r^2)$

For $r\in(-1,1)$, $r-r^2\in\left(-2, \frac{1}{4}\right]$ $\Rightarrow x\in(-8, 1]$

3. **(c) :** $\sum_{r=0}^{10}2^r\,{}^{20}C_r\,{}^{20-r}C_{10-r}$

$$=\sum_{r=0}^{10}2^r\frac{20!}{r!(20-r)!}\times\frac{(20-r)!}{10!\,(10-r)!}$$

$$=\sum_{r=0}^{10}2^r\frac{20!}{10!10!}\times\frac{10!}{r!(10-r)!}=\sum_{r=0}^{10}2^r\,{}^{20}C_{10}\,{}^{10}C_r={}^{20}C_{10}\,3^{10}$$

4. **(c) :** Given curves are $y=\ln x$ and $y=ax$

$\Rightarrow \ln x=ax$ has exactly two solutions.

$\Rightarrow \frac{\ln x}{x}=a$ has exactly two solutions.

Let $y=\frac{\ln x}{x}$, $x>0$, $\frac{dy}{dx}=\frac{x\cdot\frac{1}{x}-\ln x}{x^2}=\frac{1-\ln x}{x^2}$

y is increasing if $1-\ln x>0$, $\ln x<1$, $0<x<e$, range of y is $\left(-\infty, 1\right]$. Thus, for exactly two solutions of $\frac{\ln x}{x}=a$, $a\in\left(0, \frac{1}{e}\right)$.

5. **(b) :** Let $I=\int_0^{\pi/2}\frac{dx}{a^2\cos^2x+b^2\sin^2x}=\frac{\pi}{2ab}$

Differentiating w.r.t. a, we get

$$\int_0^{\pi/2}\frac{-2a\cos^2x\,dx}{(a^2\cos^2x+b^2\sin^2x)^2}=-\frac{\pi}{2a^2b}$$

$$\Rightarrow \int_0^{\pi/2}\frac{\cos^2x\,dx}{(a^2\cos^2x+b^2\sin^2x)^2}=\frac{\pi}{4a^3b} \quad \text{...(i)}$$

Similarly, differentiating w.r.t. b, we get

$$\int_0^{\pi/2}\frac{\sin^2x\,dx}{(a^2\cos^2x+b^2\sin^2x)^2}=\frac{\pi}{4ab^3} \quad \text{...(ii)}$$

Adding (i) & (ii), we get

$$\int_0^{\pi/2}\frac{dx}{(a^2\cos^2x+b^2\sin^2x)^2}=\frac{\pi}{4ab}\left[\frac{1}{a^2}+\frac{1}{b^2}\right]$$

$$\therefore \int_0^{\pi/2}\frac{dx}{(4\cos^2x+9\sin^2x)^2}=\frac{\pi}{24}\left[\frac{1}{4}+\frac{1}{9}\right]=\frac{13\pi}{864}$$

6. **(c) :** Putting $x=\omega, \omega^2$, we get

$(3+\omega+\omega^2)^{2010}=a_0+a_1\omega+a_2\omega^2+\ldots.$

$$\Rightarrow 2^{2010}=a_0+a_1\omega+a_2\omega^2+a_3+a_4\omega+\ldots \quad \text{...(i)}$$

$$\text{and } 2^{2010}=a_0+a_1\omega^2+a_2\omega+a_3+a_4\omega^2+\ldots \quad \text{...(ii)}$$

Adding (i) and (ii), we get

$$2^{2011}=2a_0-a_1-a_2+2a_3-a_4-a_5+2a_6-\ldots$$

$$2^{2010}=a_0-\frac{1}{2}a_1-\frac{1}{2}a_2+a_3-\frac{1}{2}a_4-\frac{1}{2}a_5+a_6-\ldots\ldots$$

7. (d) : Let mid point of AB is $M(h, k)$.
AB subtends 90° at (2, 3).
$\Rightarrow AM = MB = \sqrt{(h-2)^2+(k-3)^2}$
Also, $CM^2 + MB^2 = CB^2$
$\Rightarrow (h-2)^2 + (k-2)^2 + (h-2)^2 + (k-3)^2 = 4$
$\Rightarrow x^2+y^2-4x-5y+\frac{17}{2}=0$

8. (c) : $\left[k^2\left(\frac{\sin 3x}{3}+\frac{1}{2}\sin x\right)-\cos x-2k\sin x\right]_0^{\pi/2} \le 1$
$\Rightarrow \left\{k^2\left(\frac{-1}{3}+\frac{1}{2}\right)-0-2k\right\}-\{-1\}\le 1 \Rightarrow \frac{k^2}{6}-2k+1\le 1$
$\Rightarrow k(k-12) \le 0, 0 \le k \le 12, k = 1, 2, 3, 4, \ldots, 12.$

9. (b) : $(x\cot y+\ln\cos x)dy+(\ln\sin y-y\tan x)dx=0$
$\Rightarrow x\cot y\, dy + \ln\sin y\, dx + \ln\cos x\, dy - y\tan x\, dx = 0$
$\Rightarrow \int d(x\ln\sin y)+\int d(y\ln\cos x)=0$
$\Rightarrow x\ln\sin y + y\ln\cos x = \ln k \Rightarrow (\sin y)^x(\cos x)^y=c$

10. (d) : $\sum_{r=0}^{n-1}\left(\frac{n+1}{n}\right)\left(\frac{r.{}^nC_r\,{}^nC_{r+1}}{r+2}\right)=\sum_{r=1}^{n-1}{}^{n-1}C_{r-1}\,{}^{n+1}C_{r+2}$
$={}^{n-1}C_0\,{}^{n+1}C_3+\ldots+{}^{n-1}C_{n-2}\,{}^{n+1}C_{n+1}$
Now, $(1+x)^{n-1}={}^{n-1}C_0+{}^{n-1}C_1x+{}^{n-1}C_2x^2+\ldots+{}^{n-1}C_{n-1}x^{n-1}$
and $(x+1)^{n+1}={}^{n+1}C_0x^{n+1}+{}^{n+1}C_1x^n+{}^{n+1}C_2x^{n-1}+\ldots+{}^{n+1}C_{n+1}$
∴ Required expression
= coeff. of x^{n-2} in $(1+x)^{n-1}(x+1)^{n+1}$
= coefficient of x^{n-2} in $(1+x)^{2n}={}^{2n}C_{n-2}$

11. (b) : $|x| \le 10 \Rightarrow -10 \le x \le 10$,
$|y| \le 10 \Rightarrow -10 \le y \le 10$,
$|x-y| \le 10 \Rightarrow -10 \le x-y \le 10$
Total number of points
$= (20+1)^2 - 2[1+2+\ldots+10] = 331$

12. (a) : $f(0) = f(0) - f(0) + 2(0) - 1 = -1$
Now, $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$
$=\lim_{h\to 0}\frac{f(x)-f(h)+2xh-1-f(x)}{h}$
$=\left\{\lim_{h\to 0}\frac{-f(h)+f(0)}{h}\right\}+2x \Rightarrow f'(x)=2x-f'(0)=2x-b$
∴ $f(x) = x^2 - bx + c$
But $f(0) = -1$
$\Rightarrow f(x) = x^2 - bx - 1, D = b^2 + 4 > 0\ \forall\ b \in R$
Hence, $f(x)$ cannot be > 0 always $\forall\ x \in R$.

13. (c) : As $0<\frac{\pi}{8}<\frac{\pi}{4} \Rightarrow \cot\frac{\pi}{8}>1>\tan\frac{\pi}{8}$
$\Rightarrow a_3 > a_4$ and $a_1 > a_2$
Again, $\cot\frac{\pi}{8}>1>\tan\frac{\pi}{8}>0 \Rightarrow a_1<a_3$ and $a_1<a_4$
Hence, $a_2 < a_1 < a_4 < a_3$.

14. (a) : TM is subtangent where $T\equiv\left(x-y\frac{dx}{dy}, 0\right)$ and $M\equiv(x,0)$, $\frac{1}{2}\left(x-y\frac{dx}{dy}+x\right)=0$
$\Rightarrow 2x-y\frac{dx}{dy}=0 \Rightarrow 2\int\frac{dy}{y}=\int\frac{dx}{x}$
$\Rightarrow 2\ln y = \ln cx \Rightarrow y^2 = cx$, curve passes through (1, 1).
∴ $c = 1, y^2 = x$

15. (a) : $\vec{b}\times\vec{c}=\vec{a} \Rightarrow [\vec{a}\,\vec{b}\,\vec{c}]=\vec{a}\cdot\vec{a}=|\vec{a}|^2$
Similarly, $\bar{a}\times\bar{b}=\bar{c} \Rightarrow [\bar{a}\,\bar{b}\,\bar{c}]=|\bar{c}|^2$,
And $\vec{c}\times\vec{a}=\vec{b} \Rightarrow [\vec{a}\,\vec{b}\,\vec{c}]=|\vec{b}|^2$
Also, $[\bar{a}\,\vec{b}\,\bar{c}]=[\vec{b}\times\bar{c}\ \bar{c}\times\bar{a}\ \bar{a}\times\vec{b}]=[\bar{a}\,\vec{b}\,\bar{c}]^2 \Rightarrow [\bar{a}\,\vec{b}\,\bar{c}]=1$

16. (a) : $\cos A + \cos C = 2(1 - \cos B)$
$\Rightarrow 2\cos\frac{A+C}{2}\cos\frac{A-C}{2}=2\cdot 2\sin^2\frac{B}{2}$
$\Rightarrow 2\sin\frac{B}{2}\cos\frac{A-C}{2}\cos\frac{B}{2}=2\left(2\sin^2\frac{B}{2}\cos\frac{B}{2}\right)=2\sin B$
$\Rightarrow 2\sin\frac{A+C}{2}\cos\frac{A-C}{2}=2\sin B$
$\Rightarrow \sin A + \sin C = 2\sin B$, *i.e.* $a + c = 2b$.

17. (c) : Let $f(x) = 4x^2 - 4ax + a^2 - 2a + 2$,
vertex of the parabola is $x=\frac{a}{2}$, as $a<0 \Rightarrow \frac{a}{2}<0$
For $f(x)$ to be minimum, $f'(x) = 0 \Rightarrow x=\frac{a}{2}<0$
So, for given values $f(x)$ will be minimum at $x = 0$.
$f(0)=a^2-2a+2=3 \Rightarrow a^2-2a-1=0$
∴ $a=\frac{2\pm\sqrt{4+4}}{2}=1\pm\sqrt{2}, a=1-\sqrt{2}$

18. (b) : $\tan\frac{3\pi}{2}[x+t]=\tan\frac{3\pi}{2}[x]$
$\Rightarrow \frac{3\pi}{2}[x+t]=n\pi+\frac{3\pi}{2}[x] \Rightarrow 3([x+t]-[x])=2n$
Clearly $n = 3$, $\therefore t = 2$
$\cos^2\pi[x+t] = \cos^2\pi[x]$
$\Rightarrow \pi[x+t] = n\pi + \pi[x]\ ;\ [x+t] - [x] = n$
∴ $n = 1 \Rightarrow t = 1$ and $e^{\{x/3\}}$ is periodic with period 3.
∴ LCM of 1, 2, 3 is 6.

19. (c) : $x=\left|\dfrac{(\hat{i}+2\hat{j}+2\hat{k}).(-\hat{i}+2\hat{j}-\hat{k})}{\sqrt{1+4+1}}\right|=\dfrac{1}{\sqrt{6}}$

Now, $\cos^{-1}\cos\sqrt{6}x=\cos^{-1}\cos 1=1$

20. (d) : $y=\dfrac{\log x}{x}\Rightarrow\dfrac{dy}{dx}=-\dfrac{1}{x^2}\log x+\dfrac{1}{x}\cdot\dfrac{1}{x}$

Now, $\dfrac{dy}{dx}=0\ \Rightarrow\dfrac{1}{x^2}(1-\log x)=0$

$\Rightarrow\ \log x=1\Rightarrow x=e, x<e\ \Rightarrow\ \log x<1,\ x>e\ \Rightarrow\ \log x>1$

$\therefore$ At $x=e$, $\dfrac{dy}{dx}$ changes sign from positive to negative and hence, y has maximum value at $x = e$ and its value is $\dfrac{\log e}{e}=e^{-1}$.

21. (a) : $x = 0$ gives the maximum value.

22. (d) : Let the sides a, b, c be $\lambda, \sqrt{3}\lambda, 2\lambda$ respectively.

$c^2=a^2+b^2\quad\therefore\quad\angle C=\dfrac{\pi}{2}$

$$\cos A=\frac{b^2+c^2-a^2}{2bc}=\frac{\lambda^2(3+4-1)}{\lambda^2 4\sqrt{3}}=\frac{\sqrt{3}}{2}$$

$\therefore\ \ A=\dfrac{\pi}{6},\ B=\dfrac{\pi}{2}-A=\dfrac{\pi}{3}\ \ \therefore\ A:B:C=1:2:3$

23. (a, b) : If $\angle B+\angle D>180°\Rightarrow B$ and D lie inside the circle through other three points.

Let $\angle B+\angle D<180°$, then B and D lie outside the circle through other three points.

24. (a, c) : Since, $\sqrt{(a_1+a_2+a_3)(b_1+b_2+b_3)}$
$$=\sqrt{a_1b_1}+\sqrt{a_2b_2}+\sqrt{a_3b_3}$$

Let $a_1=A_1^2,\ a_2=A_2^2,\ a_3=A_3^2$

and $b_1=B_1^2,\ b_2=B_2^2,\ b_3=B_3^2$

$\Rightarrow (A_1^2+A_2^2+A_3^2)(B_1^2+B_2^2+B_3^2)=(A_1B_1+A_2B_2+A_3B_3)^2$

$\Rightarrow\ (A_1B_2-A_2B_1)^2+(A_2B_3-A_3B_2)^2+(A_3B_1-A_1B_3)^2=0$

$\Rightarrow\ \dfrac{A_1}{B_1}=\dfrac{A_2}{B_2}=\dfrac{A_3}{B_3}=\lambda$

or $\dfrac{\sqrt{a_1}}{\sqrt{b_1}}=\dfrac{\sqrt{a_2}}{\sqrt{b_2}}=\dfrac{\sqrt{a_3}}{\sqrt{b_3}}\ \ \therefore\ \dfrac{a_1}{b_1}=\dfrac{a_2}{b_2}=\dfrac{a_3}{b_3}$

25. (a, b, d)

26. (a, b, c, d) : $f(x+1)=\dfrac{f(x)-5}{f(x)-3}$ ....(i)

$f(x)\cdot f(x+1)-3f(x+1)=f(x)-5$

$\Rightarrow\ f(x)=\dfrac{3f(x+1)-5}{f(x+1)-1}$

Replacing x by $(x-1)$, we have

$$f(x-1)=\frac{3f(x)-5}{f(x)-1}\qquad\text{...(ii)}$$

Using (i), we have

$$f(x+2)=\frac{f(x+1)-5}{f(x+1)-3}=\frac{\dfrac{f(x)-5}{f(x)-3}-5}{\dfrac{f(x)-5}{f(x)-3}-3}=\frac{2f(x)-5}{f(x)-2}\quad\text{...(iii)}$$

Using (ii), we have

$$f(x-2)=\frac{3f(x-1)-5}{f(x-1)-1}=\frac{3\left(\dfrac{3f(x)-5}{f(x)-1}\right)-5}{\dfrac{3f(x)-5}{f(x)-1}-1}=\frac{2f(x)-5}{f(x)-2}\qquad\text{...(iv)}$$

Using (iii) and (iv), we have

$f(x+2)=f(x-2)\ \Rightarrow\ f(x+4)=f(x)$

$\Rightarrow f(x)$ is periodic with period 4.

27. (c, d) : Let $f(x)=\sec x+\operatorname{cosec}x$ for $0<x<\pi,\ x\neq\pi/2$.

$f'(x)=\sec x\tan x-\operatorname{cosec}x\cot x$

$=\sec^2x\operatorname{cosec}^2x(\sin^3x-\cos^3x)$

$\Rightarrow\ f'(x)<0$ for $0<x<\dfrac{\pi}{4}$ and

$f'(x)>0$ for $\dfrac{\pi}{4}<x<\dfrac{\pi}{2}$ and $\dfrac{\pi}{2}<x<\pi$

$\Rightarrow f(x)$ decreases strictly from ∞ to $f\left(\dfrac{\pi}{4}\right)$ for $0<x<\dfrac{\pi}{4}$ and increases from $f\left(\dfrac{\pi}{4}\right)$ to ∞ for $\dfrac{\pi}{4}<x<\dfrac{\pi}{2}$.

Also it increases strictly from $-\infty$ to ∞ for $\dfrac{\pi}{2}<x<\pi$.

Hence for any k the equation $f(x)=k$ has exactly one solution in $\left(\dfrac{\pi}{2},\pi\right)$.

If $f\left(\dfrac{\pi}{4}\right)<k<\infty$, then the equation $f(x)=k$ has exactly one solution in each of the intervals $\left(0,\dfrac{\pi}{4}\right)$ and $\left(\dfrac{\pi}{4},\dfrac{\pi}{2}\right)$. If $k<f\left(\dfrac{\pi}{4}\right)$, then $f(x)=k$ does not have any solution in $\left(0,\dfrac{\pi}{2}\right)$.

$\therefore f(x)=k$ has 3 solutions only if

$k>f\left(\dfrac{\pi}{4}\right)\ \Rightarrow\ k>2\sqrt{2}$

(28 – 30) : 28. (d) 29. (c) 30. (c)

Let $\vec{r}=x\hat{i}+y\hat{j}$, $x^2+y^2+8x-10y+40=0$,
centre $C(-4, 5)$, radius = 1, $p_1=\max\{(x+2)^2+(y-3)^2\}$
$p_2=\min\{(x+2)^2+(y-3)^2\}$
Let $P\equiv(-2, 3)$ and M be any point on circle.

$CP=2\sqrt{2}, CM=1$

$p_1=(CP+CM)^2=(2\sqrt{2}+1)^2,$

$p_2=(CP-CM)^2=(2\sqrt{2}-1)^2,$

$p_1+p_2=18$

Now, slope of $AB=\left(\frac{dy}{dx}\right)_{(2,\,2)}=-2$

Equation of AB is $2x+y=6$

$\overrightarrow{OA}=2\hat{i}+2\hat{j},\ \overrightarrow{OB}=3\hat{i},\ \overrightarrow{AB}=\hat{i}-2\hat{j},$

$\overrightarrow{AB}\cdot\overrightarrow{OB}=(\hat{i}-2\hat{j})\,(3\hat{i})=3$

31. (A) → (2), (B) → (1), (C) → (4), (D) → (3)

(A) $y=\tan^{-1}\frac{1}{2}+\tan^{-1}b,\quad (0<b<1)$

$0<\tan^{-1}\left(\frac{1+2b}{2-b}\right)\le\frac{\pi}{4}\Rightarrow 0<\left(\frac{1+2b}{2-b}\right)\le1$

$\Rightarrow 0<(1+2b)\le(2-b)$

$\Rightarrow 3b\le1\Rightarrow b\le\frac{1}{3}\Rightarrow b_{\max}=\frac{1}{3}$

(B) $\sin^4x+\cos^3x\ge1$ ...(i)
Since $\sin^2x+\cos^2x=1$ and $-1\le\sin x, \cos x\le1$
(i) can not be >1
$\therefore\ \sin^4x+\cos^3x=\pm1$ ...(ii)
(ii) is possible if either, $\sin x=\pm1$ and $\cos x=0$ or $\sin x=0$ and $\cos x=\pm1$

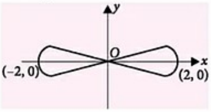
$\text{In}(0, 2\pi), x=\frac{\pi}{2},\frac{3\pi}{2}$ or $x=\pi$

$\therefore$ Number of solutions will be 3.

(C) $4y^2=x^2(4-x^2)$

$\Rightarrow y=\frac{1}{2}\sqrt{x^2(4-x^2)}\ \Rightarrow y=\frac{x}{2}\sqrt{(4-x^2)}$

$\therefore$ Area $(A)=4\times\int_0^2\frac{2x}{2\times2}\sqrt{(4-x^2)}\,dx$



Put $(4-x^2)=t\ \Rightarrow -2x\,dx=dt$

$\therefore\ A=\frac{-4}{4}\int_4^0\sqrt{t}\,dt=\int_0^4\sqrt{t}\,dt=\left[\frac{t^{3/2}}{3/2}\right]_0^4=\frac{2}{3}\times[\sqrt{64}-0]$

$\Rightarrow\ A=\frac{16}{3}$ sq. units.

(D) We have, $(1+x^2)\frac{dy}{dx}+2xy=1\Rightarrow\frac{d}{dx}[(1+x^2)y]=1$
$\Rightarrow (1+x^2)y=x+c$, Since $y(0)=0$
$\Rightarrow (1+0)0=0+c\ \Rightarrow\ c=0$

$\therefore\ (1+x^2)y=x\Rightarrow y=\frac{x}{(1+x^2)}\Rightarrow y(1)=\frac{1}{1+1}=\frac{1}{2}$

32. (A) → (2, 4), (B) → (3), (C) → (2), (D) → (5)

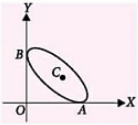
(A) The equation is $y=px+\frac{1}{p}$, where $p=\frac{dy}{dx}\ne0$,
Differentiating w.r.t. x, we get

$$p=p+x\frac{dp}{dx}-\frac{1}{p^2}\frac{dp}{dx}\ \Rightarrow\ p^2=\frac{1}{x}\ \text{ or }\ \frac{dp}{dx}=0$$

Now $\frac{dp}{dx}=0\ \Rightarrow\ p$ = constant = c

$\therefore$ Equation is $y=cx+\frac{1}{c}$, which is a straight line.

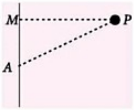
If $p^2=\frac{1}{x}$, then $y^2=\frac{1}{x}\cdot x^2+x+2x=4x$, which is a parabola.



(B) OA and OB are perpendicular tangents so O lies on the director circle and thus $OC^2=a^2+b^2$, where $2a$ is major axis and $2b$ the minor axis. Clearly, C lies on a circle.

(C) Given, $|ay-bx|=c\sqrt{(x-a)^2+(y-b)^2}$

$\Rightarrow\ \frac{|ay-bx|}{\sqrt{a^2+b^2}}=\frac{c}{\sqrt{a^2+b^2}}\sqrt{(x-a)^2+(y-b)^2}$



or $PM=kPA$, where PM is the length of perpendicular from $P(x, y)$ on the line $ay-bx=0$ and PA is the length of line segment joining P to the point $A(a, b)$ and A lies on the line, so the locus of P is a straight line through A.

(D) $hxy+ax+by+c=0\Rightarrow\ hx\left(y+\frac{a}{h}\right)+b\left(y+\frac{c}{b}\right)=0$

Above can be converted into two factors. If $\frac{a}{h}=\frac{c}{b}$, then it will represent a pair of straight lines otherwise a rectangular hyperbola.

33. **(A) → (3), (B) → (2), (C) → (1), (D) → (1)**

(A) $\vec{a}, \vec{b}$ are unit vectors.

$(\vec{a}+2\vec{b})\cdot(5\vec{a}-4\vec{b})=0 \Rightarrow 5|\vec{a}|^2+6\cdot\vec{a}\cdot\vec{b}-8|\vec{b}|^2=0$

$\Rightarrow 6\vec{a}\cdot\vec{b}=3 \Rightarrow \vec{a}\cdot\vec{b}=\frac{1}{2} \Rightarrow 2(\vec{a}\cdot\vec{b})=1$

(B) Let $\vec{a}=(1, 0, 3), \vec{b}=(-1, 3, 4), \vec{c}=(1,2,1)$ and $\vec{d}=(k, 2, 5)$

a, b, c, d are coplanar if

$[\vec{d}\,\vec{b}\,\vec{c}]+[\vec{d}\,\vec{c}\,\vec{a}]+[\vec{d}\,\vec{a}\,\vec{b}]=[\vec{a}\,\vec{b}\,\vec{c}]$

$$\Rightarrow \begin{vmatrix} k & 2 & 5 \\ -1 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix}+\begin{vmatrix} k & 2 & 5 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{vmatrix}+\begin{vmatrix} k & 2 & 5 \\ 1 & 0 & 3 \\ -1 & 3 & 4 \end{vmatrix}=\begin{vmatrix} 1 & 0 & 3 \\ -1 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix}$$

$\Rightarrow k = -1$

(C) $(1,1,m), (1,1,m+1), (1,-1,m)$ are coplanar if

$$\begin{vmatrix} 1 & 1 & m \\ 1 & 1 & m+1 \\ 1 & -1 & m \end{vmatrix}=0 \Rightarrow \begin{vmatrix} 1 & 0 & m \\ 1 & 0 & m+1 \\ 1 & -2 & m \end{vmatrix}=0$$

$\Rightarrow 2 = 0$, which is absurd.

$\therefore$ There is no value of m for which the vectors are coplanar.

(D) $\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b})\vec{c}$,

$\vec{b}\times(\vec{c}\times\vec{a})=(\vec{b}.\vec{a})\vec{c}-(\vec{c}.\vec{b})\vec{a}$ and

$\vec{c}\times(\vec{a}\times\vec{b})=(\vec{c}.\vec{b})\vec{a}-(\vec{c}.\vec{a})\vec{b}$

Since dot product is commutative, so we have, the sum = 0

34. **(A) → (2), (B) → (3), (C) → (1), (D) → (4)**

(A) $3\cdot1-2(-2)+5(\lambda)=0 \Rightarrow \lambda=-\frac{7}{5}$

(B) Point $(3, \lambda, \mu)$ lies on $2x+y+z-3=x-2y+z-1$

$\Rightarrow 3\cdot2+\lambda+\mu-3=0$ and $3-2\lambda+\mu-1=0$

$\Rightarrow \lambda+\mu+3=0$ and $2\lambda-\mu-2=0$ so $\lambda+\mu=-3$

(C) $\sin\theta=\dfrac{1\cdot4+1(-3)+1\cdot5}{\sqrt{1^2+1^2+1^2}\sqrt{16+9+25}}=\dfrac{6}{\sqrt{3}\sqrt{50}}$

$\Rightarrow \theta=\sin^{-1}\sqrt{\frac{6}{25}}$

(D) $\cos\theta=\dfrac{1\cdot3+1(-4)+1\cdot5}{\sqrt{3}\sqrt{16+9+25}}=\dfrac{4}{\sqrt{3}\sqrt{50}} \Rightarrow \theta=\cos^{-1}\sqrt{\frac{8}{75}}$

35. **(A) → (5), (B) → (2), (C) → (4), (D) → (1)**

(A) Required number of arrangements $=\dfrac{6!}{2!\,3!}-20$
$= 60 - 20 = 40$

(B) Coeff. of $\dfrac{T_{r+1}}{T_r}=\dfrac{{}^nC_r}{{}^nC_{r-1}}=\dfrac{n-r+1}{r}$ put $r = 1, 2, \ldots.$

$\frac{C_1}{C_0}=\frac{n}{1}, \frac{C_2}{C_1}=\frac{n-1}{2} \ldots,$

Now, $\left(1+\frac{C_1}{C_0}\right)\ldots\left(1+\frac{C_n}{C_{n-1}}\right)=\left(1+\frac{n}{1}\right)\left(1+\frac{n-1}{2}\right)\ldots n$ factors

$=\frac{n+1}{1}\,\frac{n+1}{2}\ldots n$ factors $=\frac{(n+1)^n}{n!}$

(C) Applying $R_1 \to R_1 + R_3 - 2R_2$, we get

$$\Delta=\begin{vmatrix} 0 & 0 & c+e-2d \\ x+2 & x+3 & x+d \\ x+3 & x+4 & x+e \end{vmatrix}$$

Given c, d, e are in A.P. $\therefore 2d = c + e$

$$\therefore \Delta=\begin{vmatrix} 0 & 0 & 0 \\ x+2 & x+3 & x+d \\ x+3 & x+4 & x+e \end{vmatrix}=0$$

(D) $c^2+d^2 \geq 2cd, d^2+e^2 \geq 2de, c^2+e^2 \geq 2ce$
[$\because$ A.M. ≥ G.M.]

$\frac{c^2+d^2+e^2}{cd+de+ce} \geq 1$.

Also in ΔCDE, $d^2+e^2-c^2=2de\cos C<2de,$

$c^2+d^2-e^2=2cd\cos E<2cd,$

$c^2+e^2-d^2=2ce\cos D<2ce$

Adding, we get

$c^2+d^2+e^2<2(ce+cd+de) \Rightarrow \frac{c^2+d^2+e^2}{ce+de+cd}<2$

36. **(4) :** $f(3)=3f(1)=3, f(4)=f(2+1+1)$
$=2+1+1=4$ and so on.

In general, we get $f(r) = r$ for $r \in N$.

$$\Rightarrow \lim_{n\to\infty}\frac{\sum_{r=1}^{n}(4r)f(3r)}{n^3}=\lim_{n\to\infty}\frac{12n(n+1)(2n+1)}{6n^3}=4$$

37. **(2) :** Put $x=r\cos\theta, y=r\sin\theta$.

$\therefore r^2\sin\theta\cos\theta = 1 - r^2 \Rightarrow r^2=\frac{2}{2+\sin2\theta}, \frac{2}{3}\leq r^2\leq2$

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Let $k = x^3y + xy^3 = r^2(1-r^2)$ and put $r^2 = t, \frac{2}{3} \le t \le 2$

So, $k = (1 - t)\,t$

$\Rightarrow \frac{dk}{dt} = 1 - 2t < 0$ for $\frac{2}{3} \le t \le 2$

So, k is min when t is max *i.e.* for $t = 2$, k is min.

When $t = 2$, min k is -2.

$\therefore\ x^3y + xy^3 + 4 \ge 2$

38. (3) : $\frac{(x-4)^2}{16} + \frac{(y-3)^2}{9} = 1$ and point P on the curve $(4(1 + \cos\theta), 3(1 + \sin\theta))$.

The reflection of the point P about the line $x - y - 2 = 0$ can be found using

$$\frac{h - 4(1+\cos\theta)}{1} = \frac{k - 3(1+\sin\theta)}{-1}$$

$$= \frac{-2(4(1+\cos\theta) - 3(1+\sin\theta) - 2)}{2} = -(4\cos\theta - 3\sin\theta) + 1$$

$h = 4(1 + \cos\theta) - 4\cos\theta + 3\sin\theta + 1 = 4 + 3\sin\theta + 1$

and $k = 3(1 + \sin\theta) + 4\cos\theta - 3\sin\theta - 1 = 3 + 4\cos\theta - 1$

$$\therefore\ \left(\frac{h-5}{3}\right)^2 + \left(\frac{k-2}{4}\right)^2 = 1$$

$\therefore$ Equation of image is $16(x - 5)^2 + 9(y - 2)^2 = 144$

$\Rightarrow 16x^2 + 9y^2 - 160x - 36y + 292 = 0$

$\therefore\ k_1 = -160,\ k_2 = 292 \ \Rightarrow\ \frac{k_1 + k_2}{44} = \frac{132}{44} = 3$

39. (2) : $5x\left(1 + \frac{1}{x^2+y^2}\right) = 12 \ \Rightarrow\ x = \frac{12}{5}\,\frac{1}{1 + \frac{1}{x^2+y^2}}$ ... (i)

Also $5y\left(1 - \frac{1}{x^2+y^2}\right) = 4 \Rightarrow y = \frac{4}{5}\,\frac{1}{1 - \frac{1}{x^2+y^2}}$ ...(ii)

Squaring (i) and (ii), and then adding we have

$$x^2 + y^2 = \left(\frac{4}{5}\right)^2 \left[\frac{9}{\left(1 + \frac{1}{x^2+y^2}\right)^2} + \frac{1}{\left(1 - \frac{1}{x^2+y^2}\right)^2}\right]$$

$$\Rightarrow\ \frac{t}{t^2} = \left(\frac{4}{5}\right)^2 \left\{\frac{9}{(t+1)^2} + \frac{1}{(t-1)^2}\right\},\ \text{where } x^2 + y^2 = t$$

$$\Rightarrow\ \frac{1}{t} = \left(\frac{4}{5}\right)^2 \left\{\frac{10t^2 - 16t + 10}{(t^2 - 1)^2}\right\}$$

$$\Rightarrow\ 25(t^4 - 2t^2 + 1) = 16t(10t^2 - 16t + 10)$$

$$\Rightarrow\ 25t^2\left(t^2 + \frac{1}{t^2} - 2\right) = 16(2t^2)\left\{5t + \frac{5}{t} - 8\right\}$$

$$\Rightarrow\ 25t^2\left(\left(t + \frac{1}{t}\right)^2 - 4\right) = 32t^2\left\{5\left(t + \frac{1}{t}\right) - 8\right\}$$

$$\Rightarrow\ 25\{z^2 - 4\} = 32\{5z - 8\},\ \text{where } t + \frac{1}{t} = z$$

$$\Rightarrow\ (5z - 6)(5z - 26) = 0 \quad \therefore\ z = \frac{6}{5} \ \text{ or } \ z = \frac{26}{5}$$

$$\Rightarrow\ t + \frac{1}{t} = \frac{6}{5} \ \text{ or } \ t + \frac{1}{t} = 5 + \frac{1}{5}$$

$\because\ t + \frac{1}{t} \ge 2$ so, either $t = 5$ or $\frac{1}{5}$

Hence, $x^2 + y^2 = 5$ or $x^2 + y^2 = \frac{1}{5}$

$$\therefore\ x = \frac{12}{5}\,\frac{1}{\left(1 + \frac{1}{5}\right)} \ \text{ or } \ x = \frac{12}{5}\left(\frac{1}{1+5}\right)$$

$$\Rightarrow\ x = \frac{12}{5}\cdot\frac{5}{6} = 2 \ \text{ or } \ x = \frac{12}{5}\cdot\frac{1}{6} = \frac{2}{5}$$

$$\Rightarrow\ y = 1 \ \text{ or } \ y = -\frac{1}{5}$$

Hence, $(x, y) = (2, 1)$ or $\left(\frac{2}{5}, -\frac{1}{5}\right)$

Hence, the given system possesses two solutions.

40. (1) : $x - y = \frac{y - z}{yz},\ y - z = \frac{z - x}{zx},\ z - x = \frac{x - y}{xy}$

Multiplying above all, we get $xyz = 1$ (As $xyz > 0$)

◆◆

COMIC CAPSULE

Why did the number 4 eat 2 turnips?

Because 2 is the square root of 4.

Exam Dates 2021
27th to 30th April; 24th to 28th May

Challenging PROBLEMS

ON

JEE Main

1. If F is the set of all onto functions from a set of vowels to a set having 3 elements and $f \in F$ is chosen randomly, then the probability that $f^{-1}(x)$ is a singleton is

(a) $\frac{7}{15}$ (b) $\frac{8}{15}$ (c) $\frac{9}{15}$ (d) $\frac{10}{15}$

2. Two persons A and B throwing 3 dice taking turns. If A throws 8, then the probability that B throws a higher number is

(a) $\frac{5}{27}$ (b) $\frac{9}{17}$ (c) $\frac{8}{27}$ (d) $\frac{20}{27}$

3. Tangents PA and PB are drawn to $y^2 = 4ax$ from P. If m_{PA} and m_{PB} are the slopes of the tangents satisfying $(m_{PA})^2 + (m_{PB})^2 = 4$, then the locus of P is

(a) $y^2 = 2x(2x + a)$ (b) $y^2 = 2x(2x - a)$
(c) $y^2 = x(x - a)$ (d) $y^2 = ax$

4. If $0 \le \arg(z) \le \frac{\pi}{4}$, then the least value of $\sqrt{2}\,|2z - 4i|$ is

(a) 6 (b) 1 (c) 4 (d) 2

5. Let $f(x) = \begin{cases} x^3 - x^2 + 10x - 5, & x \le 1 \\ -2x + \log_2(b^2 - 2), & x > 1 \end{cases}$. If $f(x)$ has greatest value at $x = 1$ and $b^2 \in (2, \lambda]$, then λ is

(a) 130 (b) 103 (c) 301 (d) 310

6. The number of four digit numbers that can be formed using the digits 1, 2, 3, 4, 5,6 that are divisible by 3, when repetition of digits is allowed, is

(a) $2^3 \times 3^2$ (b) $2^3 \times 3^3$
(c) $2^3 \times 3^4$ (d) $2^4 \times 3^3$

7. Let $f(x) = \int_{x-1}^{x+1} \frac{dt}{1+t^8}$. Then the function f

(a) has no extremum
(b) has maximum value for some $x \in (0, 1)$
(c) has minimum value for some $x \in (-1, 0)$
(d) has maximum value at $x = 0$

8. The value of $\cot\left(7\frac{1^\circ}{2}\right)$ is

(a) $\sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{6}$
(b) $\sqrt{2} + \sqrt{3} + \sqrt{5} + \sqrt{6}$
(c) $\sqrt{3} + \sqrt{4} + \sqrt{5} + \sqrt{6}$
(d) $\sqrt{2} + \sqrt{3} + \sqrt{6}$

9. If the graph of $y = ax^3 + bx^2 + cx + d$ is symmetric about the line $x = K$, then the value of $a + K$ is

(a) c (b) $c^2 - bd$
(c) $-\frac{c}{2b}$ (d) $-\frac{b}{2c}$

10. The family of curves passing through (0, 0) and satisfying the differential equation $\frac{y_2}{y_1} = 1$ (where $y_n = d^n y/dx^n$) is

(a) $y = k$ (b) $y = kx$
(c) $y = k(e^x + 1)$ (d) $y = k(e^x - 1)$

11. The solution of the differential equation $y_1 y_3 = 3y_2^2$ is

(a) $x = A_1y^2 + A_2y + A_3$ (b) $x = A_1y + A_2$
(c) $x = A_1y^2 + A_2y$ (d) None of these

12. If $I = \int_0^{\pi} x(\sin^2(\sin x) + \cos^2(\cos x))dx$, then $[I]$ = _____, where $[\cdot]$ denotes the greatest integer function.

(a) 4 (b) 3 (c) 2 (d) 1

By : Tapas Kr. Yogi (Prime Axis Classes), Visakhapatnam Mob : 09533632105, 09949428844

13. If the equation $z^4 + az^3 + bz^2 + cz + d = 0$, where a, b, c, d are real coefficients different from zero, has a purely imaginary root, then the value of the expression $\frac{c}{ab} + \frac{ad}{bc}$ equals

(a) 0 (b) 1 (c) 2 (d) 4

14. Let z be a complex number such that $\left|3z + \frac{1}{z}\right| = 1$ and $\arg(z) = \phi$, then the maximum value of $\cos^2\phi$ equals

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{1}{6}$ (d) $\frac{1}{12}$

15. The length of the perpendicular from the origin to the plane passing through the point $\vec{a}$ and containing the line $\vec{r} = \vec{b} + \lambda\vec{c}$ is

(a) $\dfrac{[\vec{a}\,\vec{b}\,\vec{c}]}{|\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a}|}$

(b) $\dfrac{[\vec{a}\,\vec{b}\,\vec{c}]}{|\vec{a}\times\vec{b} + \vec{b}\times\vec{c}|}$

(c) $\dfrac{[\vec{a}\,\vec{b}\,\vec{c}]}{|\vec{b}\times\vec{c} + \vec{c}\times\vec{a}|}$ (d) $\dfrac{[\vec{a}\,\vec{b}\,\vec{c}]}{|\vec{c}\times\vec{a} + \vec{a}\times\vec{b}|}$

16. If $((\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}))\cdot(\vec{a}\times\vec{d}) = 0$, then which of the following is always true ?

(a) $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are necessarily coplanar

(b) either $\vec{a}$ or $\vec{d}$ must lie in the plane $\vec{b}$ or $\vec{c}$

(c) either $\vec{b}$ or $\vec{c}$ must lie in plane $\vec{a}$ and $\vec{d}$

(d) either $\vec{a}$ or $\vec{b}$ must lie in plane of $\vec{c}$ and $\vec{d}$

17. Three vectors of magnitude a, $2a$, $3a$ meet in a point and their directions are along the diagonals of three adjacent faces of a cube. Then the magnitude of their resultant is

(a) $6a$ (b) $4a$ (c) $3a$ (d) $5a$

18. Let $a_0 = 1$, $a_1 = 2$ and for $n \geq 1$, $n(n + 1)a_{n+1} = n(n - 1)a_n - (n - 2)a_{n-1}$, then $\lim\limits_{n\to\infty}(a_n) =$

(a) 1 (b) 0 (c) $+\infty$ (d) e

19. If a curve represented parametrically by the equations $x = 4t^3 + 3$, $y = 4 + 3t^4$ and $\dfrac{\left(\dfrac{d^2x}{dy^2}\right)}{\left(\dfrac{dx}{dy}\right)^n}$ is a constant, then the value of n is

(a) 3 (b) 4 (c) 5 (d) 6

20. The value of $\sqrt{5 - \sqrt{10} - \sqrt{15} + \sqrt{6}}$ is

(a) $\dfrac{\sqrt{5} - \sqrt{3} - \sqrt{2}}{\sqrt{2}}$ (b) $\dfrac{\sqrt{3} + \sqrt{2} - \sqrt{5}}{\sqrt{2}}$

(c) $\dfrac{\sqrt{3} + \sqrt{2} - \sqrt{5}}{2}$ (d) $\dfrac{\sqrt{5} - \sqrt{3} - \sqrt{2}}{2}$

NUMERICAL VALUE TYPE

21. Let ABC is given triangle of side lengths $AB = 3$, $AC = 4$, $BC = 5$. Median and altitude through A cuts the side BC at D, E and circumcircle at P, Q respectively, then the value of $\frac{25}{4}\left(\frac{EQ}{DP}\right)$ is equal to _______.

22. The sum of last 3 digits of 3^{100} is _______.

23. Given ${}^8C_1x(1 - x)^7 + 2\cdot{}^8C_2x^2(1 - x)^6 + 3\cdot{}^8C_3x^3(1 - x)^5 + \dots\dots + 8\cdot x^8 = ax + b$, then $a + b =$ _______.

24. A is the $n \times n$ matrix whose elements are all '1' and B is the $n \times n$ matrix whose diagonal elements are all 'n' and other elements are '$n - r$'. Then $|(B - rI)(B - (n^2 - nr + r)I)|$ is _______.

25. If $\cos\theta + \cos^2\theta + \cos^3\theta = 1$ and $\sin^6\theta = a + b\sin^2\theta + c\sin^4\theta$, then $a + b + c =$ ______.

26. If $\tan\alpha$ is an integral solution of $4x^2 - 16x + 15 < 0$ and $\cos\beta$ is the slope of the bisector of the angle in the first quadrant between the x-axis and y-axis. Then $\sin(\alpha + \beta) : \sin(\alpha - \beta) =$ ______.

27. The sum of all the integral values of K such that the variable point $\left(K, \frac{1}{K}\right)$ remains on or inside the triangle formed by the x-axis and the tangents drawn at $(2, 1)$ on the curve $y = e^{-|x-2|}$ is _______.

28. If λ is the minimum value of the expression $|x - p| + |x - 15| + |x - p - 15|$ for x in the range $p \leq x \leq 15$ where $0 < p < 15$. Then $\frac{\lambda}{5} =$ _______.

29. Number of real roots of $2x^{99} + 3x^{98} + 2x^{97} + 3x^{96} + \dots\dots + 2x + 3 = 0$ is _______.

30. Let 'f ' is a differentiable function such that $f'(x) = f(x) + \int_0^2 f(x)\,dx$, $f(0) = \frac{4 - e^2}{3}$ then the value of $[f(2)]$, where $[\cdot]$ denotes the greatest integer $\leq x$, is _______.

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SOLUTIONS

1. (a) : Number of onto functions from set A having 5 elemetns to set B having 3 elements is 150. We shall not count onto function which satisfy $f^{-1}(x)$ is singleton. We can choose a singleton in ${}^5C_1$ ways.

The remaining 4 elements can be mapped to remaining 2 elements in $2^4 - 2 = 14$ ways.

$\therefore$ Desired probability $= \frac{5(14)}{150} = \frac{7}{15}$

2. (d) : Let E be the event that B throws a number more than 8. Then $P(E) = 1 - P(\bar{E})$

$n(\bar{E}) =$ Number of positive integral solutions of $x + y + z \le 8$

$\therefore\ n(\bar{E}) = {}^8C_3 = 56$ and $n(S) = 216\ \ \therefore P(\bar{E}) = \frac{56}{216} = \frac{7}{27}$

$\therefore$ Required Probability $= 1 - \frac{7}{27} = \frac{20}{27}$

3. (a) : Let $P \equiv (h, k)$

Equation of tangent to $y^2 = 4ax$ in slope form is

$y = mx + \frac{a}{m}$

Since, tangents pass through P.

$\therefore\ k = mh + \frac{a}{m} \Rightarrow m^2h + a - mk = 0\ \Rightarrow\ m_{PA} + m_{PB} = \frac{k}{h}$

and $m_{PA} \times m_{PB} = \frac{a}{h}$

But given that $(m_{PA})^2 + (m_{PB})^2 = 4$

$\Rightarrow\ \frac{k^2}{h^2} - \frac{2a}{h} = 4$

$\Rightarrow\ k^2 - 2ah = 4h^2$

$\therefore$ Locus of $P(h, k)$ is $y^2 = 2ax + 4x^2$

i.e., $y^2 = 2x(2x + a)$

4. (c) : The conditions cover the region bounded by x-axis and the line $y = x$. Least value of $|z - 2i|$ is the length of perpendicular from $(0, 2)$ to $y = x$ which is $\sqrt{2}$.

$\therefore$ Least value of $\sqrt{2}\,|2z - 4i| = 4$

5. (a) : We have, $\lim\limits_{x \to 1^+} f(x) \le f(1)$

$\Rightarrow -2 + \log_2(b^2 - 2) \le 5 \Rightarrow b^2 \le 130$

but $b^2 - 2 > 0\ \Rightarrow b^2 > 2$

$\therefore\ 2 < b^2 \le 130$

Hence, $\lambda = 130$

6. (d) : The first 3 digits can be filled in 6^3 ways. After filling the first 3 digits, the last digit can be filled using 1, 2, 3, 4, 5, 6 in 6 ways. But the six numbers so formed are consecutive integers out of which only two are divisible by 3. Hence, the required numbers $= 2 \times 6^3 = 2^4 \times 3^3$

7. (d) : $f(x) = \int_0^{x+1} \frac{dx}{1+x^8} - \int_0^{x-1} \frac{dx}{1+x^8}$

$$\Rightarrow\ f'(x) = \frac{1}{1+(1+x)^8} - \frac{1}{1+(1-x)^8}$$

$$= \frac{(1-x)^8 - (1+x)^8}{\{1+(1+x)^8\}\{1+(1-x)^8\}}$$

$\therefore\ f'(x) = 0 \Leftrightarrow x = 0.$

Also $x < 0 \Rightarrow f'(x) > 0$ and $x > 0 \Rightarrow f'(x) < 0$

$\therefore$ f has maximum value at $x = 0$

8. (a) : $\cot\left(7\frac{1}{2}\right)^\circ = \frac{\cos\left(7\frac{1}{2}\right)^\circ}{\sin\left(7\frac{1}{2}\right)^\circ} = \frac{2\cos^2\left(7\frac{1}{2}\right)^\circ}{\sin 15^\circ} = \frac{1+\cos 15^\circ}{\sin 15^\circ}$

Putting $\sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}, \cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$, we get

$\cot\left(7\frac{1}{2}\right)^\circ = \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{6}$

9. (c) : The graph of a cubic polynomial cannot be symmetric about $x = k$ unless $a = 0$ and then $bx^2 + cx + d$ is symmetric about $x = \frac{-c}{2b} \Rightarrow K = \frac{-c}{2b}$

$\therefore\ a + K = -\frac{c}{2b}$

10. (d) : We have, $\frac{dp}{dx} = p\left(\text{where } p = \frac{dy}{dx}\right)$

$\Rightarrow\ \ln p = x + c \Rightarrow p = e^{x+c}$

$\Rightarrow\ \frac{dy}{dx} = ke^x\ \Rightarrow\ y = ke^x + \lambda$

Since, curve passes through $(0, 0)$, so $\lambda = -k$

$\therefore\ y = k(e^x - 1)$

11. (a) : We have, $y_1y_3 = 3y_2^2$

$\Rightarrow\ \frac{y_3}{y_2} = 3\frac{y_2}{y_1} \Rightarrow \ln y_2 = 3\ln y_1 + \ln c\ \Rightarrow\ y_2 = cy_1^3$

$\Rightarrow\ \frac{y_2}{y_1^2} = cy_1\qquad \Rightarrow\ -\frac{1}{y_1} = cy + c_2$

$\Rightarrow\ \frac{dx}{dy} = -cy - c_2\ \Rightarrow\ x = -\frac{cy^2}{2} - c_2y + c_3$

$\therefore\ x = A_1y^2 + A_2y + A_3$

12. (a) : $I=\int_0^{\pi}(\pi-x)((\sin^2(\sin x)+\cos^2(\cos x))dx$

$\Rightarrow \quad 2I=2\pi\int_0^{\frac{\pi}{2}}(\sin^2(\sin x)+\cos^2(\cos x))dx$

$\Rightarrow \quad I=\pi\int_0^{\frac{\pi}{2}}(\sin^2(\sin x)+\cos^2(\cos x))dx$

$\Rightarrow \quad I=\pi\int_0^{\frac{\pi}{2}}(\sin^2(\cos x)+\cos^2(\sin x))dx$

$\Rightarrow 2I=\pi\int_0^{\frac{\pi}{2}}2dx \Rightarrow I=\frac{\pi^2}{2}=4.934$ (Approx.)

$\therefore \quad [I]=4$

13. (b) : Let ix be the root, then $ax^3-cx=0$ ...(i)
and $x^4-bx^2+d=0$ ...(ii)

From (i) $x^2=\frac{c}{a}$ and from (ii), $\frac{c^2}{a^2}-\frac{bc}{a}+d=0$

$\Rightarrow \quad \frac{c}{ab}+\frac{ad}{bc}=1$

14. (d) : Let $z=r(\cos\phi+\sin\phi)$

$\therefore \quad \left|3z+\frac{1}{z}\right|=1 \Rightarrow \left|\left(3r+\frac{1}{r}\right)\cos\phi+i\left(3r-\frac{1}{r}\right)\sin\phi\right|=1$

$\Rightarrow \quad \left(3r+\frac{1}{r}\right)^2\cos^2\phi+\left(3r-\frac{1}{r}\right)^2\sin^2\phi=1$

$\Rightarrow \quad \cos^2\phi\left\{\left(3r+\frac{1}{r}\right)^2-\left(3r-\frac{1}{r}\right)^2\right\}=1-\left(3r-\frac{1}{r}\right)^2$

$\Rightarrow \quad 12\cos^2\phi\le 1 \ \Rightarrow \cos^2\phi\le\frac{1}{12}$

15. (c) : Given plane passes through $\vec{a}$ and $\vec{b}$ containing the line is $[\vec{r}-\vec{a} \ \ \vec{b}-\vec{a} \ \ \vec{c}]=0$

$\Rightarrow \quad (\vec{r}-\vec{a})((\vec{b}-\vec{a})\times\vec{c})=0$

$\Rightarrow \quad \vec{r}\cdot(\vec{b}\times\vec{c}+\vec{c}\times\vec{a})=[\vec{a}\,\vec{b}\,\vec{c}]$

$\therefore$ Length of $\perp r$ from the origin $=\frac{[\vec{a}\,\vec{b}\,\vec{c}]}{|\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|}$

16. (c) : $((\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}))\cdot(\vec{a}\times\vec{d})=0$

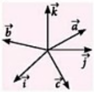
$\Rightarrow \quad ([\vec{a}\,\vec{c}\,\vec{d}]\vec{b}-[\vec{b}\,\vec{c}\,\vec{d}]\vec{a})\cdot(\vec{a}\times\vec{d})=0$

$\Rightarrow \quad [\vec{a}\,\vec{c}\,\vec{d}][\vec{b}\,\vec{a}\,\vec{d}]=0$

$\Rightarrow$ Either $\vec{c}$ or $\vec{b}$ must lie in the plane of $\vec{a}$ and $\vec{d}$.

17. (d) : Let $\vec{a},\vec{b},\vec{c}$ be three vectors, $|\vec{b}|=2a, |\vec{c}|=3a$

$\vec{a}=a\left(\frac{\hat{j}+\hat{k}}{\sqrt{2}}\right), \vec{b}=2a\left(\frac{\hat{k}+\hat{i}}{\sqrt{2}}\right), \vec{c}=3a\left(\frac{\hat{j}+\hat{i}}{\sqrt{2}}\right)$



$\therefore \quad \vec{a}+\vec{b}+\vec{c}=\frac{a}{\sqrt{2}}(5\hat{i}+4\hat{j}+3\hat{k})$

$\Rightarrow \ |\vec{a}+\vec{b}+\vec{c}|=\frac{a}{\sqrt{2}}|5\vec{i}+4\vec{j}+3\vec{k}|=5a$

18. (b) : For $n=1$, $(1\cdot2)a_2=1\Rightarrow a_2=\frac{1}{\lfloor\underline{2}}$,

For $n=2$, $(2\cdot3)a_3=2a_2-0\Rightarrow a_3=\frac{1}{2\cdot3}=\frac{1}{\lfloor\underline{3}}$

$\therefore \ a_n=\frac{1}{\lfloor\underline{n}}$ for $n\ge2$ and $\lim\limits_{n\to\infty}a_n=\lim\limits_{n\to\infty}\frac{1}{\lfloor\underline{n}}=0$

19. (c) : $\frac{dx}{dy}=\frac{dx/dt}{dy/dt}=\frac{12t^2}{12t^3}=\frac{1}{t}$

Now $\frac{d^2x}{dy^2}=\frac{d}{dy}\left(\frac{dx}{dy}\right)=\frac{d}{dt}\left(\frac{1}{t}\right)\frac{dt}{dy}=\left(\frac{-1}{t^2}\right)\frac{1}{12t^3}=\frac{-1}{12t^5}$

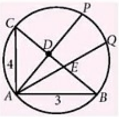
$\therefore \quad \frac{\frac{d^2x}{dy^2}}{\left(\frac{dx}{dy}\right)^n}=\frac{\left(\frac{-1}{12t^5}\right)}{\left(\frac{1}{t^n}\right)}=\frac{-t^{n-5}}{12}$, which will be constant when $n=5$

20. (b) : $5-\sqrt{10}-\sqrt{15}+\sqrt{6}$ can be written as

$\frac{3+2+5-2\sqrt{2}\sqrt{5}-2\sqrt{5}\sqrt{3}+2\sqrt{3}\sqrt{2}}{2}$

$=\left(\frac{\sqrt{3}+\sqrt{2}-\sqrt{5}}{\sqrt{2}}\right)^2$

21. (6) : For the median, $AD\cdot DP=BD\cdot DC$



$\Rightarrow \quad \frac{5}{2}\cdot DP=\frac{5}{2}\cdot\frac{5}{2} \Rightarrow DP=\frac{5}{2}$

For the altitude
$AE\cdot EQ=BE\cdot EC$

$\Rightarrow \quad \frac{12}{5}\cdot EQ=\frac{9}{5}\cdot\frac{16}{5} \Rightarrow \quad EQ=\frac{12}{5}$

$\therefore \quad \frac{25}{4}\frac{EQ}{DP}=\frac{25}{4}\times\frac{12}{5}\times\frac{2}{5}=6$

22. (1) : We have, $3^{100}=(3^4)^{25}=(81)^{25}=(80+1)^{25}$

$={}^{25}C_0\cdot(80)^{25}+{}^{25}C_1\cdot(80)^{24}+\ldots\ldots+{}^{25}C_{22}(80)^3$
$+{}^{25}C_{23}(80)^2+{}^{25}C_{24}(80)+{}^{25}C_{25}$

$=10^3[{}^{25}C_0 8^{25}\times10^{22}+{}^{25}C_1\times8^{24}\times10^{21}$
$+\ldots..+{}^{25}C_{22}\times8^3+\frac{25\times24}{2}\times(80)^2+25\times80+1$

$= 10^3m + 1920000 + 2000 + 1$, where $m \in N$

$= 10^3(m + 1920 + 2) + 1 \Rightarrow 3^{100} - 1 = 10^3(m + 1922)$

$\Rightarrow\ 3^{100} - 1$ is divisible by 1000

Thus, last three digits of 3^{100} are 001, and their sum is 1.

23. (8) : $\therefore$ Consider,

$${}^nC_1x(1-x)^{n-1} + 2\cdot{}^nC_2x^2(1-x)^{n-2} + 3\cdot{}^nC_3\,x^3(1-x)^{n-3} + \ldots.. + n\cdot{}^nC_nx^n$$

$$= \sum_{r=1}^{n} r\cdot{}^nC_rx^r\,(1-x)^{n-r} = \sum_{r=1}^{n} r\cdot\frac{n}{r}\,{}^{n-1}C_{r-1}x^r(1-x)^{n-r}$$

$$= n\sum_{r=1}^{n} {}^{n-1}C_{r-1}\,x\cdot x^{r-1}(1-x)^{(n-1)-(r-1)}$$

$$= nx\sum_{r=1}^{n} {}^{n-1}C_{r-1}\,x^{r-1}(1-x)^{(n-1)-(r-1)}$$

$= nx[x + (1 - x)]^{n-1} = nx$

$$\therefore\quad {}^8C_1(1-x)^7 + 2\cdot{}^8C_2\,x^2(1-x)^6 + 3\cdot{}^8C_3\,x^3(1-x)^5 + \ldots.. + 8\cdot{}^8C_8\,x^8$$

$= 8x = ax + b \quad \therefore\quad a + b = 8 + 0 = 8$

24. (0) : Here, $A = \begin{bmatrix} 1 & 1 & \ldots\ldots & 1 \\ 1 & 1 & \ldots\ldots & 1 \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ 1 & 1 & \ldots\ldots & 1 \end{bmatrix}$ so that

$$A^2 = A\cdot A = \begin{bmatrix} n & n & \ldots\ldots & n \\ n & n & \ldots\ldots & n \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ n & n & \ldots\ldots & n \end{bmatrix} = nA$$

Now, $B = \begin{bmatrix} n & n-r & \ldots\ldots & n-r \\ n-r & n & \ldots\ldots & n-r \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ n-r & n-r & \ldots\ldots & n \end{bmatrix}$ so that

$$B - rI = \begin{bmatrix} n-r & n-r & \ldots\ldots & n-r \\ n-r & n-r & \ldots\ldots & n-r \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ n-r & n-r & \ldots\ldots & n-r \end{bmatrix} = (n-r)A$$

Hence, $(B - rI)[B - (n^2 - nr + r)I]$

$= (B - rI)[B - rI - n(n - r)I]$

$= (n - r)^2A^2 - n(n - r)^2\,AI = (n - r)^2\,[nA - nA] = O$

$\therefore$ Required value = 0

25. (0) : We have, $\cos\theta(1 + \cos^2\theta) = \sin^2\theta$

$\Rightarrow\ (1 - \sin^2\theta)[2 - \sin^2\theta]^2 = \sin^4\theta$

$\Rightarrow\ \sin^6\theta = 4 - 8\sin^2\theta + 4\sin^4\theta$

$= a + b\sin^2\theta + c\sin^4\theta$

$\therefore\quad a = 4,\ b = -8,\ c = 4$

$\Rightarrow\quad a + b + c = 0$

26. (1) : We have, $4x^2 - 16x + 15 < 0$

$\Rightarrow\ 4x^2 - 10x - 6x + 15 < 0$

$\Rightarrow\ 2x(2x - 5) - 3(2x - 5) < 0 \Rightarrow\ (2x - 3)\,(2x - 5) < 0$

$\Rightarrow\ \dfrac{3}{2} < x < \dfrac{5}{2}$

$\therefore\quad \tan\alpha = 2$ and $\cos\beta = 1$

Now, $\dfrac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)} = \dfrac{\tan\alpha + \tan\beta}{\tan\alpha - \tan\beta} = \dfrac{2+0}{2-0} = 1$

27. (2) : We have, $y = e^{-|x-2|}$ and a variable point $\left(K, \dfrac{1}{K}\right)$.

Range of K is given by intersection of tangents at (2, 1) with $xy = 1$ which is

$$\frac{1+\sqrt{5}}{2} \le K \le \frac{3+\sqrt{5}}{2} \Rightarrow K = 2$$

28. (3) : $|x - p| = x - p$ (since $x \ge p$)

$|x - 15| = 15 - x$ (since $x \le 15$)

$|x - (p + 15)| = (p + 15) - x$ (since, $15 + p > x$)

$\therefore$ Given expression reduces to

$E = x - p + 15 - x + p + 15 - x$

$\Rightarrow\ E = 30 - x$

$\therefore\ E_{\min}$ occurs when $x = 15$

$\Rightarrow\ \lambda = 15 \Rightarrow\ \dfrac{\lambda}{5} = 3$

29. (1) : Given equation can be written as

$$(2x+3)(x^{98} + x^{96} \ldots.. + x^2 + 1) = (2x+3)\frac{(x^{100}-1)}{x^2-1}$$

So, the real roots are $x = \pm 1, \dfrac{-3}{2}$, out of which ±1 are not the roots of given equation.

30. (5) : Given $f'(x) = f(x) + A$, where $A = \int_0^2 f(x)dx$

After solving, we get

$$f(x) = \lambda(e^x - 1) + \frac{4-e^2}{3}$$

$$\because\ \int_0^2 f(x)dx = A \Rightarrow \lambda = 1 \text{ and } A = \frac{e^2-1}{3}$$

$$\therefore\ f(x) = e^x - 1 + \frac{4-e^2}{3} = e^x - \frac{1}{3}(e^2 - 1)$$

$$\Rightarrow\ f(2) = \frac{2e^2+1}{3} \quad \therefore\ [f(2)] = 5$$

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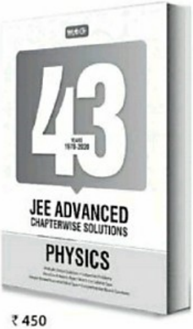
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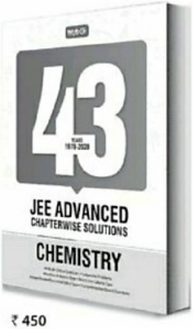
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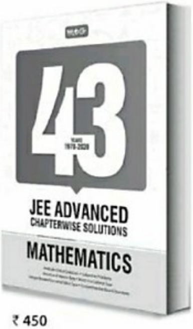
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YOU ASK
WE ANSWER

Do you have a question that you just can't get answered? Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough. The best questions and their solutions will be printed in this column each month.

1. There are n intermediate stations on a railway line from one terminus to another. In how many ways can the train stop at 3 of these intermediate stations if

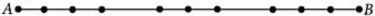
(i) all the three stations are consecutive?

(ii) at least two of the stations are consecutive?

(iii) no two of these stations are consecutive?

(Ritu Mehta, Delhi)

Ans.



(i) The number of triplets of consecutive stations, viz. $S_1S_2S_3$, $S_2S_3S_4$, $S_3S_4S_5$, ..., $S_{n-2}S_{n-1}S_n$ is $(n-2)$.

(ii) The total number of consecutive pair of stations, viz. S_1S_2, S_2S_3, ..., $S_{n-1}\,S_n$ is $(n-1)$.

Each of the above pair can be associated with a third station in $(n-2)$ ways. Thus, choosing a pair of stations and any third station can be done in $(n-1)(n-2)$ ways. The above count also includes the case of three consecutive stations. However, we can see that each such case has been counted twice. For example, the pair S_4S_5 combined with S_6 and the pair S_5S_6 combined with S_4 are identical.

Hence, subtracting the excess counting, the number of ways in which three stations can be chosen so that at least two of them are consecutive is

$$(n-1)(n-2)-(n-2)=(n-2)^2.$$

(iii) Without restriction, the train can stop at any three stations in ${}^nC_3$ ways. Hence, the number of ways the train can stop so that no two stations are consecutive is

$${}^nC_3-(n-2)^2=\frac{n(n-1)(n-2)}{1\cdot2\cdot3}-(n-2)^2$$

$$=(n-2)\left(\frac{n^2-n-6n+12}{6}\right)=\frac{(n-2)(n-3)(n-4)}{6}={}^{n-2}C_3$$

2. P, Q and R are the feet of the normals drawn to a parabola $(y-3)^2=8(x-2)$. A circle cuts the above parabola at points P, Q, R and S. Then this circle always passes through the point

(a) (2, 3) (b) (3, 2)

(c) (0, 3) (d) (2, 0)

(Prateush Sharma, Jamshedpur)

Ans. (a) : Given equation of parabola is

$(y-3)^2=8(x-2)$

Here, vertex of parabola is (2, 3)

We know that, a circle passes through three co-normal points of a parabola always passes through the vertex of the parabola.

∴ The circle through P, Q, R and S out of which P, Q, R are co-normal points, will always passes through (2, 3) *i.e.*, vertex of parabola.

3. Find the values of a, b, c if the following limit exists :

$$\lim_{x\to0}\frac{ae^x-b\ln(1+x)+cxe^{-x}}{x^2\sin x}=2$$

(Arjun, Gujarat)

Ans. Let $L=\lim\limits_{x\to0}\dfrac{ae^x-b\ln(1+x)+cxe^{-x}}{x^2\sin x}$

$$=\lim_{x\to0}\frac{ae^x+cxe^{-x}-b\ln(1+x)}{x^3}\cdot\frac{x}{\sin x}$$

$$=\lim_{x\to0}\frac{ae^x+cxe^{-x}-b\ln(1+x)}{x^3}$$

$$=\lim_{x\to0}\frac{a\left(1+x+\frac{x^2}{2!}+\ldots\right)+cx\left(1-x+\frac{x^2}{2!}-\ldots\right)-b\left(x-\frac{x^2}{2}+\frac{x^3}{3}+\ldots\right)}{x^3}$$

$$=\lim_{x\to0}\frac{a+(a+c-b)x+(a-2c+b)\frac{x^2}{2}+\left(\frac{a}{3!}+\frac{c}{2!}-\frac{b}{3}\right)x^3+\ldots}{x^3}$$

For a finite non-zero limit to exist, we have

$a=0$ ...(i) $a+c-b=0$ ...(ii)

$a-2c+b=0$ ...(iii)

and $\dfrac{a}{6}+\dfrac{c}{2}-\dfrac{b}{3}=2$ [According to the given condition]

i.e. $a+3c-2b=12$ ...(iv)

Solving (i), (ii), (iii) and (iv), we can see that there is no value of a, b, c which satisfies all the equations.

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MONTHLY TEST DRIVE

XI

This specially designed column enables students to self analyse their extent of understanding of all chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80 **Time Taken : 60 Min.**

Only One Option Correct Type

1. There are three piles of identical red, blue and green balls and each pile contains at least 10 balls. The number of ways of selecting 10 balls if twice as many red balls as green balls are to be selected is

(a) 5 (b) 4 (c) 3 (d) 2

2. If the slopes of the lines $3x^2 + 2hxy + 4y^2 = 0$ are in the ratio 3 : 1, then h equals

(a) $\frac{1}{4}$ (b) ± 4 (c) $-\frac{1}{2}$ (d) $-\frac{1}{4}$

3. If $\sin^4 x + \cos^4 y + 2 = 4 \sin x \cos y$ and $0 \le x, y \le \frac{\pi}{2}$, then $\sin x + \cos y$ is equal to

(a) -2 (b) 0 (c) 2 (d) $\frac{3}{2}$

4. The coefficient of x^{n-2} in the polynomial $(x-1)(x-2)(x-3) \dots (x-n)$ is

(a) $\frac{n(n^2+2)(3n+1)}{24}$ (b) $\frac{n(n^2-1)(3n+2)}{24}$

(c) $\frac{n(n^2+1)(3n+4)}{24}$ (d) none of these

5. Different words are being formed by arranging the letters of the word "SUCCESS". All the words obtained by writing in the form of a dictionary. The number of words in which the two C are together but no two S are together, is

(a) 120 (b) 96 (c) 24 (d) 420

6. If the sides of a right angled triangle form an A.P., then the sines of the acute angles are

(a) $\frac{3}{5}, \frac{4}{5}$ (b) $\sqrt{3}, \frac{1}{3}$

(c) $\sqrt{\left(\frac{\sqrt{5}-1}{2}\right)}, \sqrt{\left(\frac{\sqrt{5}+1}{2}\right)}$

(d) $\frac{\sqrt{3}}{2}, \frac{1}{2}$

One or More Than One Option(s) Correct Type

7. If α, β are the roots of $ax^2 + bx + c = 0$ and $\alpha + h$, $\beta + h$ are the roots of $px^2 + qx + r = 0$, then

(a) $h = \frac{1}{2}\left(\frac{b}{a} - \frac{q}{p}\right)$ (b) $\frac{b^2 - 4ac}{a^2} = \frac{q^2 - 4pr}{p^2}$

(c) $\frac{a}{p} = \frac{b}{q} = \frac{c}{r}$ (d) None of these

8. A light ray coming along the line $3x + 4y = 5$ gets reflected from the line $ax + by = 1$ and goes along the line $5x - 12y = 10$. Then,

(a) $a = 14/15, b = 112/15$
(b) $a = 14/15, b = -8/115$
(c) $a = 64/115, b = -8/115$
(d) $a = 64/15, b = 14/15$

9. Two circles each of radius 5 units touch each other at (1, 2). If equation of their common tangent is $4x + 3y = 10$, then the equations of the circles are

(a) $x^2 + y^2 + 4x + 4y - 25 = 0$
(b) $x^2 + y^2 + 6x + 2y - 15 = 0$
(c) $x^2 + y^2 - 10x - 10y + 25 = 0$
(d) $x^2 + y^2 + 8x + 4y + 15 = 0$

10. If p and q are two statements, then $\sim (p \wedge q) \vee \sim (q \Leftrightarrow p)$ is

(a) tautology (b) contradiction
(c) neither tautology nor contradiction
(d) either tautology or contradiction

11. Three lines $x+2y+3=0$; $x+2y-7=0$ and $2x-y-4=0$ form 3 sides of two squares, then the equation of remaining sides of these squares are

(a) $2x-y+6=0$ (b) $x-y+2=0$
(c) $2x-y-14=0$ (d) $x-y-14=0$

12. If $\lim\limits_{x\to\infty}\left(1+\frac{\lambda}{x}+\frac{\mu}{x^2}\right)^{2x}=e^2$, then

(a) $\lambda=1, \mu=2$ (b) $\lambda=2, \mu=1$
(c) $\lambda=1$, μ = any real constant
(d) $\lambda=\mu=1$

13. The common tangents to the circle $x^2+y^2=8$ and the parabola $y^2=16x$ are

(a) $x-y+4=0$ (b) $x+y+4=0$
(c) $2x+3y+6=0$ (d) $2x-3y-6=0$

Comprehension Type

If m, n, r are positive integers and if $r<m, r<n$, then
${}^mC_r+{}^mC_{r-1}\,.\,{}^nC_1+{}^mC_{r-2}\,.\,{}^nC_2+\ldots+{}^nC_r$
= Coefficient of x^r in $(1+x)^m(1+x)^n$
= Coefficient of x^r in $(1+x)^{m+n}={}^{m+n}C_r$

14. If $S_n={}^nC_0\,.\,{}^nC_1+{}^nC_1\,.\,{}^nC_2+\ldots+{}^nC_{n-1}\,.\,{}^nC_n$ and if $\frac{S_{n+1}}{S_n}=\frac{15}{4}$, then n equals

(a) 2, 4 (b) 4, 6 (c) 6, 8 (d) 8, 10

15. If $(1+x)^n=C_0+C_1x+C_2x^2+\ldots+C_nx^n$ and n is odd, then the value of $C_0^2-C_1^2+C_2^2-C_3^2+\ldots+(-1)^nC_n^2$ is

(a) 0 (b) ${}^{2n}C_n$
(c) $(-1)^n\,{}^{2n}C_{n-1}$ (d) ${}^{2n}C_{n-2}$

Matrix Match Type

16. Match the following.

	Column I	Column II	
P.	If $x_1, x_2, \ldots, x_{18}$ are observations such that $\sum\limits_{j=1}^{18}(x_j-8)=9$ and $\sum\limits_{j=1}^{18}(x_j-8)^2=45$, then the standard deviation of these observations is	1.	25
Q.	The number of solutions of the equation $2\cos\left(\frac{x}{2}\right)=3^x+3^{-x}$ is	2.	1.5
R.	If odds against solving a question by three students are 2 : 1, 5 : 2 and 5 : 3 respectively, then probability that the question is solved only by one student in simplest form is $\frac{m}{n}$, then m equals	3.	1
S.	If $f(x)=\frac{x-4}{2\sqrt{x}}$, then $f'(1)$ is	4.	1.25

	P	Q	R	S
(a)	1	3	4	2
(b)	2	3	1	4
(c)	3	1	2	4
(d)	3	2	4	1

Numerical Value Type

17. If $z^2+z+1=0$, then $\sum\limits_{r=1}^{6}\left(z^6+\frac{1}{z^r}\right)^2=$ _______.

18. The foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ and the hyperbola $\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}$ coincide. Then, the value of b^2 is _______.

19. A pair of dice is thrown thrice. The probability of throwing doublets at least once is m, then value of $\frac{216}{13}m$ is _______.

20. If $a_i>0, i=1,2,3,\ldots,50$ and $a_1+a_2+a_3+\ldots+a_{50}=50$, then the minimum value of $\frac{1}{25}\left(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+\ldots+\frac{1}{a_{50}}\right)$ is equal to _______.

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Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ……
No. of questions correct ……
Marks scored in percentage ……

Check your score! If your score is

> 90%	**EXCELLENT WORK !**	You are well prepared to take the challenge of final exam.
90-75%	**GOOD WORK !**	You can score good in the final exam.
74-60%	**SATISFACTORY !**	You need to score more next time.
< 60%	**NOT SATISFACTORY!**	Revise thoroughly and strengthen your concepts.

Exam on 31st May 2021

CBSE warm-up!

CLASS-XII

Practice paper for CBSE Exam as per the reduced syllabus and marking scheme issued by CBSE for the academic session 2020-21.

Practice Paper 2021

Time Allowed : 3 hours
Maximum Marks : 80

GENERAL INSTRUCTIONS

(1) This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks.
(2) Part-A has Objective Type Questions and Part -B has Descriptive Type Questions.
(3) Both Part A and Part B have internal choices.

Part-A :

(1) It consists of two sections- I and II.
(2) Section I comprises of 16 very short answer type questions.
(3) Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

Part-B :

(1) It consists of three sections- III, IV and V.
(2) Section III comprises of 10 questions of 2 marks each.
(3) Section IV comprises of 7 questions of 3 marks each.
(4) Section V comprises of 3 questions of 5 marks each.
(5) Internal choice is provided in 3 questions of Section-III, 2 questions of Section-IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.

PART-A

SECTION-I

All questions are compulsory. In case of internal choices attempt any one.

1. Evaluate: $\int xe^{x^2}\,dx$

OR

Evaluate : $\int \frac{\cos x}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^3}\,dx$

2. Let $f : N \to N$ be defined by $f(n)=\begin{cases}\frac{n+1}{2}, & \text{if } n \text{ is odd}\\ \frac{n}{2}, & \text{if } n \text{ is even}\end{cases}$ for all $n \in N$. Find whether the function f is bijective or not.

3. A matrix A of order 3×3 has determinant 5. What is the value of $|3A|$?

OR

If $f(x)=\begin{vmatrix}(1+x)^{17} & (1+x)^{19} & (1+x)^{23}\\ (1+x)^{23} & (1+x)^{29} & (1+x)^{34}\\ (1+x)^{41} & (1+x)^{43} & (1+x)^{47}\end{vmatrix}$

$= A + Bx + Cx^2 + \ldots$, then prove that $A = 0$.

4. Find the direction cosines of the line joining A(0, 7, 10) and B(−1, 6, 6).

5. Solve the differential equation $\dfrac{dy}{dx}=2^{y-x}$.

OR

Solve the differential equation $\dfrac{dy}{dx}=\left(\dfrac{y}{x}\right)^{1/3}$.

6. Prove that the function $f(x)=\sqrt{3}\,\sin 2x-\cos 2x+4$ is one-one in the interval $\left[-\dfrac{\pi}{6},\dfrac{\pi}{3}\right]$.

7. A bag contains 5 red and 3 blue balls. If 3 balls are drawn at random without replacement, then find the probability of getting exactly one red ball.

OR

If $P(A)=\dfrac{2}{5}, P(B)=\dfrac{3}{10}$ and $P(A\cap B)=\dfrac{1}{5}$, then find the value of $P(A' \mid B')$.

8. If a matrix has 12 elements, then how many orders are possible?

9. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitude 1 and 2 respectively, such that $\vec{a}\cdot\vec{b}=1$.

OR

Find the projection of $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$ on $\vec{b}=\hat{i}-2\hat{j}+\hat{k}$.

10. If $P(A)=\dfrac{7}{13}, P(B)=\dfrac{9}{13}$ and $P(A\cap B)=\dfrac{4}{13}$, then find the value of $P(A \mid B)$.

11. If E and F are events such that $0 < P(F) < 1$, then prove that $P(E\mid F)+P(\bar{E}\mid F)=1$.

12. Check whether the function $f: N \to N$ defined by $f(x) = 4 - 3x$ is one-one or not.

13. A die, whose faces are marked 1, 2, 3 in red and 4, 5, 6 in green, is tossed. Let A be the event "number obtained is even" and B be the event "number obtained is red". Find $P(A \cap B)$.

14. Find the area enclosed between the curve $x^2+y^2=16$ and the coordinate axes in the first quadrant.

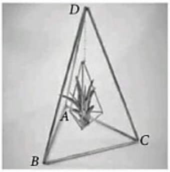
15. Write a 3 × 2 matrix whose elements in the i^{th} row and j^{th} column are given by $a_{ij}=\dfrac{(2i-j)}{2}$.

16. Find the direction cosines of the line passing through two points (2, 1, 0) and (1, −2, 3).

SECTION - II

Both the case study based questions are compulsory. Attempt any 4 sub-parts from each question 17 and 18. Each sub-part carries 1 mark.

17. Ginni purchased an air plant holder which is in the shape of a tetrahedron. Let A, B, C and D are the coordinates of the air plant holder, where $A\equiv(1,1,1), B\equiv(2,1,3)$, $C \equiv (3, 2, 2)$ and $D \equiv (3, 3, 4)$.



Based on the above information, answer the following questions.

(i) Find the position vector of $\overrightarrow{AB}$.

(a) $-\hat{i}-2\hat{k}$ (b) $2\hat{i}+\hat{k}$

(c) $\hat{i}+2\hat{k}$ (d) $-2\hat{i}-\hat{k}$

(ii) Find the position vector of $\overrightarrow{AC}$.

(a) $2\hat{i}-\hat{j}-\hat{k}$ (b) $2\hat{i}+\hat{j}+\hat{k}$

(c) $-2\hat{i}-\hat{j}+\hat{k}$ (d) $\hat{i}+2\hat{j}+\hat{k}$

(iii) Find the position vector of $\overrightarrow{AD}$.

(a) $2\hat{i}-2\hat{j}-3\hat{k}$ (b) $\hat{i}+\hat{j}-3\hat{k}$

(c) $3\hat{i}+2\hat{j}+2\hat{k}$ (d) $2\hat{i}+2\hat{j}+3\hat{k}$

EXAM ALERT 2021

Exam	Date
JEE Main	27th to 30th April; 24th to 28th May
SRMJEEE	11th to 16th June
VITEEE	18th to 26th June
COMEDK (Engg.)	20th June
BITSAT	24th to 30th June
JEE Advanced	3rd July
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CBSE CHAMPION Chapterwise -Topicwise Solved Papers Series contains topicwise questions and solutions asked over last decade in CBSE-Board examination.

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(iv) Area of $\Delta ABC =$

(a) $\dfrac{\sqrt{11}}{2}$ sq. units (b) $\dfrac{\sqrt{14}}{2}$ sq. units

(c) $\dfrac{\sqrt{13}}{2}$ (d) $\dfrac{\sqrt{17}}{2}$ sq. units

(v) Find the unit vector along $\overrightarrow{AD}$.

(a) $\dfrac{1}{\sqrt{17}}(2\hat{i}+2\hat{j}+3\hat{k})$

(b) $\dfrac{1}{\sqrt{17}}(3\hat{i}+3\hat{j}+2\hat{k})$

(c) $\dfrac{1}{\sqrt{11}}(2\hat{i}+2\hat{j}+3\hat{k})$ (d) $(2\hat{i}+2\hat{j}+3\hat{k})$

18. Western music concert is organised every year in the stadium that can hold 36000 spectators. With ticket price of ₹ 10, the average attendance has been 24000. Some financial expert estimated that price of a ticket should be determined by the function $p(x)=15-\dfrac{x}{3000}$, where x is the number of tickets sold.

Based on the above information, answer the following questions.

(i) The revenue, R as a function of x can be represented as

(a) $15x-\dfrac{x^2}{3000}$ (b) $15-\dfrac{x^2}{3000}$

(c) $15x-\dfrac{1}{30000}$ (d) $15x-\dfrac{x}{3000}$

(ii) The range of x is

(a) [24000, 36000] (b) [0, 24000]

(c) [0, 36000] (d) none of these

(iii) The value of x for which revenue is maximum, is

(a) 20000 (b) 21000

(c) 22500 (d) 25000

(iv) When the revenue is maximum, the price of the ticket is

(a) ₹ 5 (b) ₹ 5.5

(c) ₹ 7 (d) ₹ 7.5

(v) How many spectators should be present to maximize the revenue?

(a) 21500 (b) 21000

(c) 22000 (d) 22500

PART-B

SECTION-III

All questions are compulsory. In case of internal choices attempt any one.

19. Find the equation of normal to the curve $x^{\frac{2}{3}}+y^{\frac{2}{3}}=2$ at (1, 1).

20. A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
$P(X)$	0	K	$2K$	$2K$	$3K$	K^2	$2K^2$	$7K^2+K$

Determine :

(i) K (ii) $P(X<3)$

OR

If $P(\text{not } A)=0.7$, $P(B)=0.7$ and $P(B \mid A)=0.5$, then find $P(A \mid B)$ and $P(A \cup B)$.

21. Evaluate : $\displaystyle\int_0^1 \frac{x\tan^{-1}x}{(1+x^2)^{3/2}}\,dx$

22. Find the principal value of $\cot^{-1}\left[\dfrac{\sqrt{1-\sin\frac{\pi}{2}}+\sqrt{1+\sin\frac{\pi}{2}}}{\sqrt{1-\sin\frac{\pi}{2}}-\sqrt{1+\sin\frac{\pi}{2}}}\right]$.

23. Evaluate : $\displaystyle\int \frac{10x^9+10^x\log_e 10}{10^x+x^{10}}\,dx$

OR

Evaluate : $\displaystyle\int \frac{1}{\sin x+\sqrt{3}\cos x}\,dx$

24. Solve the differential equation $(x-1)\dfrac{dy}{dx}=2xy$.

OR

Solve the differential equation $5\dfrac{dy}{dx}=e^x y^4$.

25. If $A=\begin{bmatrix}1 & 3\\ 3 & 10\end{bmatrix}$, then find A^{-1}.

26. A coin is tossed and then a die is thrown. Find the probability of obtaining a '6' given that head came up.

27. If $y=\cot x$, then show that $\dfrac{d^2y}{dx^2}+2y\dfrac{dy}{dx}=0$.

28. The x-coordinate of a point on the line joining the points $P(2, 2, 1)$ and $Q(5, 1, -2)$ is 4. Find its z-coordinate.

SECTION-IV

All questions are compulsory. In case of internal choices attempt any one.

29. Let A and B be non-empty sets. Show that $f : A \times B \to B \times A$ such that $f(a, b) = (b, a)$ is a bijective function.

30. If $f(x) = a \log |x| + bx^2 + x$ has extreme values at $x = -1$ and $x = 2$, then find a and b.

31. Find the solution of the differential equation $y^2\,dx + (xy + x^2)\,dy = 0$.

OR

Find the particular solution of $\ln\left(\frac{dy}{dx}\right) = 3x + 4y,\ y(0) = 0.$

32. If $f(x)$ is continuous at $x = 0$, where

$$f(x) = \begin{cases} \dfrac{\sin x}{x} + \cos x, & \text{for } x > 0 \\ \dfrac{4(1-\sqrt{1-x})}{x}, & \text{for } x < 0 \end{cases}$$, then find $f(0)$.

33. Evaluate : $\displaystyle\int_1^2 \frac{dx}{x(1+x^2)}$

OR

Evaluate : $\int \sin x \sin 2x \sin 3x\,dx$

34. Find the area bounded by the circle $x^2 + y^2 = 8x$ and the line $x = 2$.

35. If $y = (x)^{\cos x} + (\cos x)^{\sin x}$, then find $\frac{dy}{dx}$.

SECTION-V

All questions are compulsory. In case of internal choices attempt any one.

36. Solve the following problem graphically :
Maximize $Z = 22x + 18y$
Subject to constraints :
$x + y \le 20$
$36x + 24y \le 576$
$x, y \ge 0$

OR

Find the maximum value of $z = 3x + 5y$
subject to constraints :
$x + 4y \le 24,\ 3x + y \le 21,\ x + y \le 9,\ x \ge 0,\ y \ge 0.$

37. Find the equation of the plane containing the lines $\vec{r} = \hat{i} + \hat{j} + \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and $\vec{r} = \hat{i} + \hat{j} + \mu(-\hat{i} + \hat{j} - 2\hat{k})$. Find the distance of this plane from origin and also from the point $(1, 1, 1)$.

OR

Find the equation of the plane which contains the line of intersection of the planes $x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$ and whose x-intercept is twice its z-intercept.

38. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, then find A^{-1} and hence solve the system of linear equations $x + 2y + z = 4$, $-x + y + z = 0$, $x - 3y + z = 2$.

OR

Prove that $\begin{vmatrix} yz-x^2 & zx-y^2 & xy-z^2 \\ zx-y^2 & xy-z^2 & yz-x^2 \\ xy-z^2 & yz-x^2 & zx-y^2 \end{vmatrix}$ is divisible by $(x + y + z)$, and hence find the quotient.

SOLUTIONS

1. Let $I = \int x e^{x^2}\,dx$

Put $x^2 = t \Rightarrow 2x\,dx = dt \Rightarrow x\,dx = \frac{dt}{2}$

$$\therefore\quad I = \frac{1}{2}\int e^t\,dt = \frac{e^t}{2} + C = \frac{e^{x^2}}{2} + C$$

OR

$$\text{Let } I = \int \frac{\cos x}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^3}dx = \int \frac{\cos^2(x/2) - \sin^2(x/2)}{\{\cos(x/2) + \sin(x/2)\}^3}dx$$

$$= \int \frac{\cos(x/2) - \sin(x/2)}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2}dx$$

Put $t = \cos\frac{x}{2} + \sin\frac{x}{2} \Rightarrow dt = \frac{1}{2}\left(-\sin\frac{x}{2} + \cos\frac{x}{2}\right)dx$

$$\therefore\quad I = 2\int \frac{dt}{t^2} = \frac{-2}{t} + C = \frac{-2}{\cos(x/2) + \sin(x/2)} + C$$

2. Here, $f(1) = \frac{1+1}{2} = 1,\ f(2) = \frac{2}{2} = 1,$

$$f(3) = \frac{3+1}{2} = 2,\ f(4) = \frac{4}{2} = 2$$

Thus, $f(2k-1) = \dfrac{(2k-1)+1}{2} = k$ and

$$f(2k) = \frac{2k}{2} = k$$

$\Rightarrow$ $f(2k-1) = f(2k)$, where $k \in N$
But, $2k - 1 \neq 2k \Rightarrow f$ is not one-one.
Hence, f is not bijective.

3. Given, $|A| = 5$, order of A is 3×3.
$\therefore$ $|3A| = 3^3|A| = 27 \times 5 = 135$

OR

Given that,

$$f(x) = \begin{vmatrix} (1+x)^{17} & (1+x)^{19} & (1+x)^{23} \\ (1+x)^{23} & (1+x)^{29} & (1+x)^{34} \\ (1+x)^{41} & (1+x)^{43} & (1+x)^{47} \end{vmatrix}$$

$$= A + Bx + Cx^2 + ...$$

On putting $x = 0$, we have $f(0) = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = A$

$\Rightarrow$ $A = 0$ $\qquad$ ($\because$ R_1 and R_2 are identical)

4. Direction ratios of AB are
$(-1 - 0, 6 - 7, 6 - 10)$ or $(-1, -1, -4)$

Also, $\sqrt{(-1)^2 + (-1)^2 + (-4)^2} = 3\sqrt{2}$

$\therefore$ Direction cosines are $\left(-\dfrac{1}{3\sqrt{2}}, -\dfrac{1}{3\sqrt{2}}, \dfrac{-4}{3\sqrt{2}}\right)$

or $\left(\dfrac{1}{3\sqrt{2}}, \dfrac{1}{3\sqrt{2}}, \dfrac{4}{3\sqrt{2}}\right)$

5. We have, $\dfrac{dy}{dx} = 2^{y-x} \Rightarrow \dfrac{dy}{2^y} = \dfrac{dx}{2^x}$

Integrating both sides, we get

$$\frac{-2^{-y}}{\log 2} = \frac{-2^{-x}}{\log 2} + C$$

$\Rightarrow$ $-2^{-y} + 2^{-x} = C\log 2 = k(\text{say}) \Rightarrow 2^{-x} - 2^{-y} = k$

OR

We have, $\dfrac{dy}{dx} = \left(\dfrac{y}{x}\right)^{1/3} \Rightarrow y^{-1/3}\,dy = x^{-1/3}\,dx$

Integrating both sides, we get

$$\frac{3}{2}y^{2/3} = \frac{3}{2}x^{2/3} + k \Rightarrow y^{2/3} - x^{2/3} = \frac{2}{3}k = c\ (\text{say})$$

Hence, required solution is $y^{2/3} - x^{2/3} = c$

6. We have, $f(x) = 2\left(\dfrac{\sqrt{3}}{2}\sin 2x - \dfrac{1}{2}\cos 2x\right) + 4$

$$= 2\left(\cos\frac{\pi}{6}\sin 2x - \sin\frac{\pi}{6}\cos 2x\right) + 4$$

$\Rightarrow$ $f(x) = 2\sin\left(2x - \dfrac{\pi}{6}\right) + 4$

$\because$ $\sin x$ is one-one in $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$.

$\therefore$ $-\dfrac{\pi}{2} \le 2x - \dfrac{\pi}{6} \le \dfrac{\pi}{2} \Rightarrow x \in \left[-\dfrac{\pi}{6}, \dfrac{\pi}{3}\right]$

7. Required probability $= P\{(RBB), (BRB), (BBR)\}$
$= P(RBB) + P(BRB) + P(BBR)$

$$= \frac{5}{8}\times\frac{3}{7}\times\frac{2}{6} + \frac{3}{8}\times\frac{5}{7}\times\frac{2}{6} + \frac{3}{8}\times\frac{2}{7}\times\frac{5}{6} = 3\times\frac{5}{56} = \frac{15}{56}$$

OR

Given, $P(A) = \dfrac{2}{5}, P(B) = \dfrac{3}{10}, P(A \cap B) = \dfrac{1}{5}$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{2}{5} + \frac{3}{10} - \frac{1}{5} = \frac{1}{2}$$

$\therefore$ $P(A' \cap B') = P((A \cup B)') = 1 - P(A \cup B) = 1 - \dfrac{1}{2} = \dfrac{1}{2}$

Also, $P(B') = 1 - P(B) = 1 - \dfrac{3}{10} = \dfrac{7}{10}$

$\therefore$ $P(A' | B') = \dfrac{P(A' \cap B')}{P(B')} = \dfrac{1/2}{7/10} = \dfrac{5}{7}$

8. All possible orders of 12 elements are
$1 \times 12, 12 \times 1, 2 \times 6, 6 \times 2, 3 \times 4, 4 \times 3$ *i.e.*, 6.

9. We are given $|\vec{a}| = 1, |\vec{b}| = 2$ and $\vec{a}\cdot\vec{b} = 1$

Now, we have $\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \dfrac{1}{1\times 2} = \dfrac{1}{2}$

or $\theta = \cos^{-1}\left(\dfrac{1}{2}\right) = \dfrac{\pi}{3}$

OR

We have, $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$

Projection of $\vec{a}$ on $\vec{b}$ is $\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

$$\vec{a}\cdot\vec{b} = (2\hat{i} - \hat{j} + \hat{k})\cdot(\hat{i} - 2\hat{j} + \hat{k}) = 2 + 2 + 1 = 5$$

$$|\vec{b}|=\sqrt{1^2+(-2)^2+1^2}=\sqrt{1+4+1}=\sqrt{6}$$

$\therefore$ Projection of $\vec{a}$ on $\vec{b}$ is $\frac{5}{\sqrt{6}}$.

10. We have, $P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{\frac{4}{13}}{\frac{9}{13}}=\frac{4}{9}$

11. $P(E|F)+P(\bar{E}|F)$

$$=\frac{P(E\cap F)+P(\bar{E}\cap F)}{P(F)}=\frac{P((E\cup\bar{E})\cap F)}{P(F)}=\frac{P(F)}{P(F)}=1$$

12. We have, $f: N\rightarrow N, f(x)=4-3x$

Let $f(x_1)=f(x_2)\Rightarrow 4-3x_1=4-3x_2 \Rightarrow x_1=x_2$

$\therefore$ f is one-one.

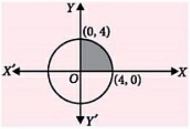
13. We have, $S=\{1, 2, 3, 4, 5, 6\}$

Let A be the event that number is even $=\{2, 4, 6\}$ and B be the event that number is red $=\{1, 2, 3\}$

Now, $A\cap B=\{2\}$

$\therefore$ $P(A\cap B)=\frac{1}{6}$

14. Given curve is a circle with centre (0, 0) and radius 4.



$\therefore$ Required area

$$=\int_0^4\sqrt{16-x^2}\,dx$$

$$=\left[\frac{x}{2}\sqrt{16-x^2}+\frac{16}{2}\sin^{-1}\frac{x}{4}\right]_0^4=4\pi\text{ sq. units}$$

15. $a_{ij}=\frac{2i-j}{2}$, $i=1, 2, 3$ and $j=1, 2$

$$a_{11}=\frac{2-1}{2}=\frac{1}{2},\ a_{12}=\frac{2-2}{2}=0,$$

$$a_{21}=\frac{2(2)-1}{2}=\frac{3}{2},\ a_{22}=\frac{2(2)-2}{2}=\frac{2}{2}=1,$$

$$a_{31}=\frac{2(3)-1}{2}=\frac{5}{2},\ a_{32}=\frac{2(3)-2}{2}=\frac{4}{2}=2$$

$$\therefore\ \text{Required matrix}=\begin{bmatrix}\frac{1}{2} & 0\\ \frac{3}{2} & 1\\ \frac{5}{2} & 2\end{bmatrix}$$

16. Here, $P(2, 1, 0)$ and $Q(1, -2, 3)$

So, $PQ=\sqrt{(1-2)^2+(-2-1)^2+(3-0)^2}$

$=\sqrt{1+9+9}=\sqrt{19}$

Thus, the direction cosines of the line joining two points are $<\frac{1-2}{\sqrt{19}},\frac{-2-1}{\sqrt{19}},\frac{3-0}{\sqrt{19}}>$ *i.e.*, $<\frac{-1}{\sqrt{19}},\frac{-3}{\sqrt{19}},\frac{3}{\sqrt{19}}>$

17. (i) (c) : Position vector of $\overline{AB}$

$=(2-1)\hat{i}+(1-1)\hat{j}+(3-1)\hat{k}=\hat{i}+2\hat{k}$

(ii) (b) : Position vector of $\overline{AC}$

$=(3-1)\hat{i}+(2-1)\hat{j}+(2-1)\hat{k}=2\hat{i}+\hat{j}+\hat{k}$

(iii) (d) : Position vector of $\overline{AD}$

$=(3-1)\hat{i}+(3-1)\hat{j}+(4-1)\hat{k}=2\hat{i}+2\hat{j}+3\hat{k}$

(iv) (b) : Area of $\Delta ABC=\frac{1}{2}|\overline{AB}\times\overline{AC}|$

$$\overline{AB}\times\overline{AC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 0 & 2\\ 2 & 1 & 1\end{vmatrix}=\hat{i}(0-2)-\hat{j}(1-4)+\hat{k}(1-0)$$

$$=-2\hat{i}+3\hat{j}+\hat{k}$$

$$\Rightarrow\ |\overline{AB}\times\overline{AC}|=\sqrt{(-2)^2+3^2+1^2}$$

$$=\sqrt{4+9+1}=\sqrt{14}$$

$\therefore$ Area of $\Delta ABC=\frac{1}{2}\sqrt{14}$ sq. units

(v) (a) : Unit vector along $\overline{AD}=\frac{\overline{AD}}{|\overline{AD}|}$

$$=\frac{2\hat{i}+2\hat{j}+3k}{\sqrt{2^2+2^2+3^2}}=\frac{2\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{4+4+9}}=\frac{1}{\sqrt{17}}(2\hat{i}+2\hat{j}+3\hat{k})$$

18. (i) (a) : Let p be the price per ticket and x be the number of tickets sold.

Then, revenue function $R(x)=p\times x=\left(15-\frac{x}{3000}\right)x$

$$=15x-\frac{x^2}{3000}$$

(ii) (c) : Since, more than 36000 tickets cannot be sold. So, range of x is [0, 36000].

(iii) (c) : We have, $R(x)=15x-\frac{x^2}{3000}$

$\Rightarrow$ $R'(x)=15-\frac{x}{1500}$

For maxima/minima, put $R'(x)=0$

$\Rightarrow$ $15-\frac{x}{1500}=0\Rightarrow x=22500$

Also, $R''(x)=-\frac{1}{1500}<0.$

(iv) **(d) :** Maximum revenue will be at $x = 22500$

$\therefore$ Price of a ticket $= 15-\frac{22500}{3000} = 15 - 7.5 =$ ₹ 7.5

(v) **(d) :** Number of spectators will be equal to number of tickets sold.

$\therefore$ Required number of spectators = 22500

19. Differentiating $x^{2/3} + y^{2/3} = 2$ with respect to x, we get

$$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\,\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\left(\frac{y}{x}\right)^{1/3}$$

$\therefore$ Slope of the tangent at (1, 1) = – 1

Also, the slope of the normal at (1, 1) is given by

$$\frac{-1}{\text{slope of the tangent at (1, 1)}} = 1$$

Therefore, the equation of the normal at (1, 1) is

$y - 1 = 1(x - 1) \Rightarrow y - x = 0$

20. (i) Since $\Sigma P(X) = 1$

$\therefore\ 0 + K + 2K + 2K + 3K + K^2 + 2K^2 + 7K^2 + K = 1$

$\Rightarrow\ 10K^2 + 9K - 1 = 0$

$$\Rightarrow\ K = \frac{-9\pm\sqrt{81+40}}{20} = \frac{-9\pm 11}{20} = \frac{1}{10}, -1$$

Since the probability of the event lies between 0 and 1.

So, $K = \frac{1}{10}$.

(ii) $P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$

$$= 0 + K + 2K = 3K = \frac{3}{10} \qquad \left(\because K = \frac{1}{10}\right)$$

OR

We have, $P(\text{not } A) = 0.7$ or $P(\bar{A}) = 0.7$

$\Rightarrow\ 1 - P(A) = 0.7 \Rightarrow P(A) = 0.3 \quad \left[\because P(A) + P(\bar{A}) = 1\right]$

Now, $P(B|A) = \frac{P(A\cap B)}{P(A)}$

$$\Rightarrow\ 0.5 = \frac{P(A\cap B)}{0.3} \Rightarrow P(A\cap B) = 0.15$$

$$\therefore\ P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{0.15}{0.7} = \frac{3}{14}$$

and $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.3 + 0.7 - 0.15 = 0.85$

21. Let $\tan^{-1} x = \theta \Rightarrow x = \tan\theta \Rightarrow dx = \sec^2\theta\, d\theta$

Now, $x = 0 \Rightarrow \theta = 0$ and $x = 1 \Rightarrow \theta = \frac{\pi}{4}$

$$\therefore\ I = \int_0^1 \frac{x\tan^{-1}x}{(1+x^2)^{3/2}}dx = \int_0^{\pi/4}\frac{\theta\tan\theta}{\sec^3\theta}\sec^2\theta\, d\theta$$

$$= \int_0^{\pi/4}\theta\sin\theta\, d\theta = [-\theta\cos\theta]_0^{\pi/4} - \int_0^{\pi/4}(-\cos\theta)\, d\theta$$

$$= [-\theta\cos\theta]_0^{\pi/4} + [\sin\theta]_0^{\pi/4} = \frac{4-\pi}{4\sqrt{2}}$$

22. We have, $\cot^{-1}\left[\frac{\sqrt{1-\sin\frac{\pi}{2}}+\sqrt{1+\sin\frac{\pi}{2}}}{\sqrt{1-\sin\frac{\pi}{2}}-\sqrt{1+\sin\frac{\pi}{2}}}\right]$

$$= \cot^{-1}\left[\frac{0+\sqrt{2}}{0-\sqrt{2}}\right] \qquad \left[\because \sin\frac{\pi}{2} = 1\right]$$

$= \cot^{-1}(-1)$

$$= \frac{3\pi}{4} \qquad \left[\because \cot\frac{3\pi}{4} = \cot\left(\pi - \frac{\pi}{4}\right) = -\cot\frac{\pi}{4} = -1 \text{ and } \frac{3\pi}{4} \in (0, \pi)\right]$$

23. Let $I = \int\frac{10x^9 + 10^x\log_e 10}{10^x + x^{10}}dx$

Put $10^x + x^{10} = t \Rightarrow (10^x \log_e 10 + 10x^9)dx = dt$

$$\therefore\ I = \int\frac{dt}{t} = \log_e t + c = \log_e(10^x + x^{10}) + c$$

OR

We have,

$$I = \int\frac{1}{\sin x + \sqrt{3}\cos x}\,dx = \frac{1}{2}\int\frac{dx}{\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x}$$

$$I = \frac{1}{2}\int\frac{1}{\sin\left(x+\frac{\pi}{3}\right)}dx = \frac{1}{2}\int\operatorname{cosec}\left(x+\frac{\pi}{3}\right)dx$$

$$I = \frac{1}{2}\log\left|\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)\right| + C$$

24. We have, $(x - 1)\frac{dy}{dx} = 2xy$

$$\Rightarrow\ (x-1)dy = 2xy\,dx \Rightarrow \frac{dy}{dx} = \frac{2xy}{x-1}$$

$$\Rightarrow\ \int\frac{1}{y}dy = 2\int\frac{x}{x-1}\,dx = 2\int\frac{x-1+1}{x-1}dx$$

$\Rightarrow\ \log|y| + C = 2\,[x + \log|x - 1|]$

OR

We have, $5\frac{dy}{dx} = e^x y^4 \Rightarrow \frac{5dy}{y^4} = e^x dx$

On integrating both sides, we get

$$5.\int y^{-4}dy=\int e^{x}dx \Rightarrow 5.\frac{y^{-3}}{(-3)}=e^{x}+C \Rightarrow \frac{-5}{3y^{3}}=e^{x}+C$$

25. Let $A=\begin{bmatrix}1 & 3\\ 3 & 10\end{bmatrix}$

$\therefore \quad |A|=10-9=1\neq 0$

So, A^{-1} exists.

Now, adj $A=\begin{bmatrix}10 & -3\\ -3 & 1\end{bmatrix}$

$$\Rightarrow \quad A^{-1}=\frac{1}{|A|}(\text{adj}\,A)=\begin{bmatrix}10 & -3\\ -3 & 1\end{bmatrix}.$$

26. The sample space S associated to the given random experiment is given by

$S=\{(H,1),(H,2),(H,3),(H,4),(H,5),(H,6),(T,1),(T,2),(T,3),(T,4),(T,5),(T,6)\}$ and

Let the event $B=\{(H,6),(T,6)\}$ and

$A=\{(H,1),(H,2),(H,3),(H,4),(H,5),(H,6)\}$

$\therefore$ Required probability $=P(B\mid A)$

$$=\frac{P(A\cap B)}{P(A)}=\frac{\frac{1}{12}}{\frac{6}{12}}=\frac{1}{6}$$

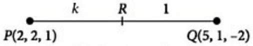
27. We have, $y=\cot x \Rightarrow \dfrac{dy}{dx}=-\operatorname{cosec}^2 x$

$$\Rightarrow \frac{dy}{dx}=-(1+\cot^2 x) \Rightarrow \frac{dy}{dx}=-(1+y^2)$$

Differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2}=-\left(2y\frac{dy}{dx}\right) \Rightarrow \frac{d^2y}{dx^2}+2y\frac{dy}{dx}=0$$

28. Given that $P(2,2,1)$ and $Q(5,1,-2)$



Let the point R on the line PQ, divides the line in the ratio $k:1$ and x-coordinate of point R on the line is 4.

So, by section formula

$$4=\frac{5k+2}{k+1} \Rightarrow k=2$$

Now, z-coordinate of point R,

$$z=\frac{-2k+1}{k+1}=\frac{-2\times 2+1}{2+1}=-1$$

$\Rightarrow$ z-coordinate of point $R=-1$

29. Injectivity : Let (a_1,b_1) and $(a_2,b_2)\in A\times B$ such that, $f(a_1,b_1)=f(a_2,b_2)$

$\Rightarrow \quad (b_1,a_1)=(b_2,a_2) \Rightarrow b_1=b_2$ and $a_1=a_2$

$\Rightarrow \quad (a_1,b_1)=(a_2,b_2)$

Thus, $f(a_1,b_1)=f(a_2,b_2) \Rightarrow (a_1,b_1)=(a_2,b_2)$ for all $(a_1,b_1),(a_2,b_2)\in A\times B$.

So, f is an injective function.

Surjectivity : Let (b,a) be an arbitrary element of $B\times A$, where, $b\in B$ and $a\in A \Rightarrow (a,b)\in A\times B$

Thus, for all $(b,a)\in B\times A$, their exists $(a,b)\in(A\times B)$ such that, $f(a,b)=(b,a)$

So, $f: A\times B\to B\times A$ is surjective function.

Hence, f is a bijective function.

30. Clearly, domain $(f)=R-\{0\}$

We have, $f(x)=a\log|x|+bx^2+x \Rightarrow f'(x)=\dfrac{a}{x}+2bx+1$

Since $f(x)$ has extreme values at $x=-1$ and $x=2$.

Therefore, $f'(-1)=0$ and $f'(2)=0$

$\Rightarrow \quad -a-2b+1=0$ and $\dfrac{a}{2}+4b+1=0$

$\Rightarrow \quad a+2b=1$ ...(i)

and $\quad a+8b=-2$ ...(ii)

Solving (i) and (ii), we get $a=2$ and $b=-\dfrac{1}{2}$.

31. We have, $y^2dx+(xy+x^2)dy=0 \Rightarrow \dfrac{dy}{dx}=\dfrac{-y^2}{xy+x^2}$

Put $y=vx \Rightarrow \dfrac{dy}{dx}=v+x\dfrac{dv}{dx}$

$$\therefore \quad v+x\frac{dv}{dx}=\frac{-v^2x^2}{vx^2+x^2} \Rightarrow x\frac{dv}{dx}=\frac{-v^2}{v+1}-v$$

$$\Rightarrow \quad x\frac{dv}{dx}=-\left(\frac{2v^2+v}{v+1}\right) \Rightarrow \int\left(\frac{v+1}{v(2v+1)}\right)dv=-\int\frac{dx}{x}$$

$$\Rightarrow \quad \int\left(\frac{1}{v}-\frac{1}{2v+1}\right)dv=-\log x+\log c$$

$$\Rightarrow \quad \log v-\frac{1}{2}\log|2v+1|+\log x=\log c$$

$$\Rightarrow \quad \log\left|\frac{v^2x^2}{2v+1}\right|=\log c^2 \Rightarrow \frac{v^2x^2}{2v+1}=c^2$$

$$\Rightarrow \quad y^2=c^2\left(\frac{2y}{x}+1\right) \Rightarrow xy^2=c^2(x+2y)$$

OR

We have, $\ln\left(\dfrac{dy}{dx}\right)=3x+4y$

$$\Rightarrow \quad \frac{dy}{dx}=e^{3x}\cdot e^{4y} \Rightarrow e^{-4y}dy=e^{3x}dx$$

On integration, we get $-\dfrac{1}{4}e^{-4y}=\dfrac{e^{3x}}{3}+C$

At $x=0, y=0$; we have $-\dfrac{1}{4}=\dfrac{1}{3}+C \Rightarrow C=-\dfrac{7}{12}$

$\therefore$ Solution is $\dfrac{e^{-4y}}{4}+\dfrac{e^{3x}}{3}=\dfrac{7}{12} \Rightarrow 3e^{-4y}+4e^{3x}=7$

32. Since, $f(x)$ is continuous at $x=0$, therefore

$$f(0)=\lim_{x\to 0^-} f(x)=\lim_{x\to 0^+} f(x) \qquad \ldots\text{(i)}$$

$$\lim_{x\to 0^-} f(x)=\lim_{x\to 0}\left(\frac{4(1-\sqrt{1-x})}{x}\right)$$

$$=4\lim_{x\to 0}\left(\frac{1-(1-x)}{x(1+\sqrt{1-x})}\right)=4\lim_{x\to 0}\left(\frac{1-1+x}{x(1+\sqrt{1-x})}\right)$$

$$=4\left(\frac{1}{1+1}\right)=\frac{4}{2}=2$$

From (i), we get $f(0)=2$

33. Let $I=\displaystyle\int_1^2 \frac{dx}{x(1+x^2)}$

Consider, $\dfrac{1}{x(1+x^2)}=\dfrac{A}{x}+\dfrac{Bx+C}{1+x^2}$

$\Rightarrow 1=A(1+x^2)+(Bx+C)\cdot x \Rightarrow 1=x^2(A+B)+Cx+A$

On equating the coefficient of x^2, x and the constant term from both sides, we get $A=1$, $B=-1$ and $C=0$

$$\therefore\quad I=\int_1^2 \frac{1}{x}\,dx+\int_1^2 \frac{-x}{1+x^2}\,dx$$

$$=\left[\log x\right]_1^2-\int_1^2 \frac{x}{1+x^2}\,dx=\log 2-\int_1^2 \frac{x\,dx}{1+x^2}$$

Put $1+x^2=t \Rightarrow 2x\,dx=dt$

When $x=1$, $t=2$ and when $x=2$, $t=5$

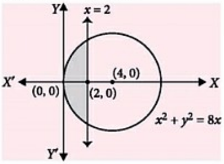
$$\therefore\quad I=\log 2-\frac{1}{2}\int_2^5 \frac{1}{t}\,dt=\log 2-\frac{1}{2}\left[\log t\right]_2^5$$

$$=\log 2-\frac{1}{2}[\log 5-\log 2]=\log 2-\frac{1}{2}\log\left(\frac{5}{2}\right)=\frac{1}{2}\log\left(\frac{8}{5}\right)$$

OR

Refer to Answer 26, page no. 156 of MTG CBSE Champion Mathematics, Class 12.

34. Given curve is $x^2+y^2=8x$



$$\therefore\quad \text{Required area}=2\int_0^2 \sqrt{8x-x^2}\,dx$$

$$=2\int_0^2 \sqrt{(4)^2-(x-4)^2}\,dx$$

$$=2\left[\left(\frac{x-4}{2}\right)\sqrt{4^2-(x-4)^2}+\frac{16}{2}\sin^{-1}\left(\frac{x-4}{4}\right)\right]_0^2$$

$$=2\left[\left(\frac{-2}{2}\right)\sqrt{16-4}+8\sin^{-1}\left(\frac{-1}{2}\right)-8\sin^{-1}(-1)\right]$$

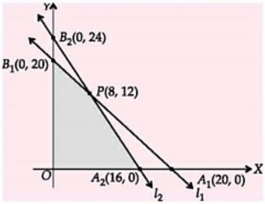
$$=2\left[-\sqrt{12}-8\times\frac{\pi}{6}+\frac{8\pi}{2}\right]=2\left[-\sqrt{12}+\frac{16\pi}{6}\right]$$

$$=\left(\frac{16\pi}{3}-2\sqrt{12}\right)\text{sq. units}$$

35. *Refer to Answer 48, page no. 100-101 of MTG CBSE Champion Mathematics, Class 12.*

36. To solve this LPP graphically, we first convert the inequations into equations $l_1: x+y=20$, $l_2: 36x+24y=576$, $l_3: x=0$, $l_4: y=0$. Draw the corresponding lines.

The feasible region of the LPP is shaded in figure. The corner points of the feasible region OA_2PB_1 are $O(0,0)$, $A_2(16,0)$, $P(8,12)$ and $B_1(0,20)$.



The values of the objective function Z at corner-points of the feasible region are given in the following table.

Corner Points	Value of $Z=22x+18y$
$O(0,0)$	0
$A_2(16,0)$	352
$P(8,12)$	392 (maximum)
$B_1(0,20)$	360

$\therefore$ Z has maximum value 392 at $P(8,12)$.

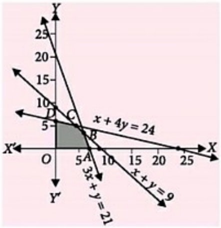
OR

Converting inequations into equations and drawing the corresponding lines.

$x+4y=24$, $3x+y=21$, $x+y=9$

i.e., $\dfrac{x}{24}+\dfrac{y}{6}=1$, $\dfrac{x}{7}+\dfrac{y}{21}=1$, $\dfrac{x}{9}+\dfrac{y}{9}=1$

As $x\geq 0$, $y\geq 0$ solution lies in first quadrant



The feasible region of the LPP is shaded in figure. The corner points of the feasible region *OABCD* are $O(0, 0)$, $A(7, 0)$, $B(6, 3)$, $C(4, 5)$ and $D(0, 6)$.

Corner Points	Value of $Z = 3x + 5y$
$O(0, 0)$	0
$A(7, 0)$	21
$B(6, 3)$	33
$C(4, 5)$	37 (Maximum)
$D(0, 6)$	30

$\therefore$ Z has maximum value 37 at $C(4, 5)$.

37. We are given the equation of lines as

$\vec{r} = \hat{i} + \hat{j} + \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and

$\vec{r} = \hat{i} + \hat{j} + \mu(-\hat{i} + \hat{j} - 2\hat{k})$

We know that the equation of plane containing $\vec{r} = \vec{a} + \lambda\vec{b}_1$ and $\vec{r} = \vec{a} + \lambda\vec{b}_2$ is $(\vec{r} - \vec{a})\cdot(\vec{b}_1 \times \vec{b}_2) = 0$

Here, $\vec{a} = \hat{i} + \hat{j}$, $\vec{b}_1 = \hat{i} + 2\hat{j} - \hat{k}$, $\vec{b}_2 = -\hat{i} + \hat{j} - 2\hat{k}$

Now,

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ -1 & 1 & -2 \end{vmatrix} = (-4 + 1)\hat{i} - (-2 - 1)\hat{j} + (1 + 2)\hat{k}$$

$$\Rightarrow \quad \vec{b}_1 \times \vec{b}_2 = -3\hat{i} + 3\hat{j} + 3\hat{k}$$

Therefore, the equation of plane is,

$$(\vec{r} - (\hat{i} + \hat{j}))\cdot(-3\hat{i} + 3\hat{j} + 3\hat{k}) = 0$$

$$\Rightarrow \vec{r}\cdot(-3\hat{i} + 3\hat{j} + 3\hat{k}) + 3 - 3 = 0 \Rightarrow \vec{r}\cdot(-\hat{i} + \hat{j} + \hat{k}) = 0$$

This is the equation of the required plane.

We know that the distance of a point P with position vector $\vec{a}$ from the plane $\vec{r}\cdot\vec{n} = d$ is given by $\dfrac{|\vec{a}\cdot\vec{n} - d|}{|\vec{n}|}$.

Now, distance from origin is 0 and distance from the point $(1, 1, 1)$ *i.e.*, $\hat{i} + \hat{j} + \hat{k}$ is

$$\frac{|(\hat{i} + \hat{j} + \hat{k})\cdot(-\hat{i} + \hat{j} + \hat{k})|}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{|-1 + 1 + 1|}{\sqrt{3}} = \frac{1}{\sqrt{3}} \text{ units}$$

OR

Refer to Answer 70, page no. 271 of MTG CBSE Champion Mathematics, Class 12.

38. We have, $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$

$$\therefore \quad |A| = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix}$$

$= 1(1 + 3) + 1(2 + 3) + 1(2 - 1) = 10 \neq 0$

So, A is invertible.

$$\therefore \quad \text{adj } A = \begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}' = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$\Rightarrow \quad A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix} \qquad \text{... (i)}$$

Now, the given system of equations is expressible as

$$\begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

or $\quad A'X = B$, where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$

Now, $|A'| = |A| = 10 \neq 0$. So, the given system of equations has a solution given by

$X = (A')^{-1}B = (A^{-1})'B \qquad [\because (A')^{-1} = (A^{-1})']$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}'\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} \qquad \text{[Using (i)]}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 9/5 \\ 2/5 \\ 7/5 \end{bmatrix}$$

$\Rightarrow \quad x = 9/5,\ y = 2/5$ and $z = 7/5$

OR

Refer to Answer 63, page no. 66 of MTG CBSE Champion Mathematics, Class 12.

❖❖

MONTHLY TEST DRIVE

XII

This specially designed column enables students to self analyse their extent of understanding of all chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80 **Time Taken : 60 Min.**

Only One Option Correct Type

1. The largest interval lying in $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$ for which the function, $f(x)=4^{-x^2}+\cos^{-1}\left(\frac{x}{2}-1\right)+\log(\cos x)$ is defined, is

(a) $\left[-\frac{\pi}{4},\frac{\pi}{2}\right)$ (b) $\left[0,\frac{\pi}{2}\right)$

(c) $[0, \pi]$ (d) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

2. The value of $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix}$ is

(a) 0 (b) 1

(c) 2 (d) none of these

3. In which of the following functions, langranges mean value theorem is NOT applicable in [0, 1]?

(a) $f(x)=\begin{cases} \frac{1}{2}-x, & x<\frac{1}{2} \\ \left(\frac{1}{2}-x\right)^2, & x\ge\frac{1}{2} \end{cases}$

(b) $f(x)=\begin{cases} \frac{\sin x}{x}, & x\ne 0 \\ 1, & x=0 \end{cases}$

(c) $f(x) = x|x|$ (d) $f(x) = |x|$

4. The projection of the line $\frac{x+1}{-1}=\frac{y}{2}=\frac{z-1}{3}$ on the plane $x-2y+z=6$ is the line of intersection of this plane with the plane

(a) $2x+y+2=0$ (b) $3x+y-z=2$

(c) $2x-3y+8z=3$ (d) none of these

5. The locus of all the points on the curve $y^2=4a\left(x+a\sin\left(\frac{x}{a}\right)\right)$ at which the tangent is parallel to x-axis is

(a) a straight line (b) a circle

(c) a parabola (d) an ellipse

6. If $\frac{d^2x}{dy^2}\left(\frac{dy}{dx}\right)^3+\frac{d^2y}{dx^2}=k$, then k is equal to

(a) 0 (b) 1

(c) 2 (d) None of these

One or More Than One Option(s) Correct Type

7. Let $f : R \rightarrow R$ be a function defined by $f(x)=\frac{e^{|x|}-e^{-x}}{e^x+e^{-x}}$, then $f(x)$ is

(a) not one-one (b) not onto

(c) onto but not one-one

(d) one-one but not onto

8. The probabilities that a student passes in Mathematics, Physics and Chemistry are m, p and c respectively. Of these subjects, a student has a 75% chance of passing in atleast one, 50% chance of passing in atleast two and 40% chance of passing in exactly two subjects, then which of the following relation(s) is (are) true?

(a) $p+m+c=\frac{19}{20}$ (b) $p+m+c=\frac{27}{20}$

(c) $pmc=\frac{1}{10}$ (d) $pmc=\frac{1}{4}$

9. If P, Q, R are angles of triangle PQR, then the value of $\begin{vmatrix} -1 & \cos R & \cos Q \\ \cos R & -1 & \cos P \\ \cos Q & \cos P & -1 \end{vmatrix}$ is equal to

(a) -1 (b) 0 (c) $\frac{1}{2}$ (d) 1

10. If $\int \frac{(2x-1)dx}{x^4-2x^3+x+1} = A\tan^{-1}\left[\frac{f(x)}{\sqrt{3}}\right]+C$, then

(a) $A=\frac{2}{\sqrt{3}}$ (b) $A = 2$

(c) $f(x) = 2x^2 - 2x - 1$ (d) $f(x) = 2x^2 + 2x + 1$

11. Non-zero vectors $\vec{a}$ and $\vec{b}$ are equally inclined to vector $\vec{c}$, if $\vec{c}$ is equal to

(a) $\frac{|\vec{b}|}{|\vec{a}|+|\bar{b}|}\bar{a}+\frac{|\vec{a}|}{|\vec{a}|+|\bar{b}|}\bar{b}$

(b) $\frac{|\vec{a}|}{|\vec{a}|+|\bar{b}|}\bar{a}+\frac{|\vec{b}|}{|\vec{a}|+|\bar{b}|}\bar{b}$

(c) $\frac{|\vec{a}|}{|\vec{a}|+2|\bar{b}|}\bar{a}+\frac{|\vec{b}|}{|\vec{a}|+2|\bar{b}|}\bar{b}$

(d) $\frac{|\vec{b}|}{2|\vec{a}|+|\bar{b}|}\bar{a}+\frac{|\vec{a}|}{2|\vec{a}|+|\bar{b}|}\bar{b}$

12. Let $f(x)=\begin{cases}\int_0^x (1+|1-t|)dt, \ x>2 \\ 5x+1, x\le 2\end{cases}$, then

(a) $f(x)$ is not continuous at $x = 2$
(b) $f(x)$ is continuous but not differentiable at $x = 2$
(c) $f(x)$ is differentiable everywhere
(d) the right derivative of $f(x)$ at $x = 2$ does not exist.

13. If $D^* f(x)=\lim\limits_{h\to 0}\frac{f^2(x+h)-f^2(x)}{h}$, where $f^2(x)=\{f(x)\}^2$

The value of $D^* f(x)$ at the point on the curve $y = f(x)$ such that tangent to it are parallel to x-axis, then

(a) $f(x)$ (b) 0 (c) $2f(x)$ (d) $xf(x)$

Comprehension Type

Let two planes $P_1 : 2x - y + z = 2$ and $P_2 : x + 2y - z = 3$ are given.

14. The equation of the acute angle bisector of planes P_1 and P_2 is

(a) $x - 3y + 2z + 1 = 0$ (b) $3x + y - 5 = 0$

(c) $x + 3y - 2z + 1 = 0$ (d) $3x + z + 7 = 0$

15. The Image of plane P_1 in the plane mirror P_2 is

(a) $x + 7y - 4x + 5 = 0$ (b) $3x + 4y - 5z + 9 = 0$

(c) $7x - y + 2z - 9 = 0$ (d) None of the above

Matrix Match Type

16. Match the following.

	Column I		Column II
P.	Let $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$, then	1.	$f(x)$ increases, if $x \in (2, 3)$
Q.	Let $f(x) = 2 \ln(x-2) - x^2 + 4x + 1$, then	2.	$f(x)$ increases, if $x \in (0, 1)\cup(2, \infty)$
R.	Let $f(x)=\frac{\lvert x-1\rvert}{x^2}$, then	3.	$f(x)$ increases, if $x \in (1, 2)\cup(3, \infty)$
		4.	$f(x)$ decreases, if $x \in (0, 1)\cup(2, \infty)$

	P	Q	R
(a)	2	1	3
(b)	1	3	4
(c)	3	1	4
(d)	3	4	1

Numerical Value Type

17. If $f : R \to R$, $g : R \to R$ and $h : R \to R$ is such that $f(x) = x^2$, $g(x) = \tan x$ and $h(x) = \log x$, then the value of $[ho(gof)](x)$, if $x=\frac{\sqrt{\pi}}{2}$ will be ______.

18. The vectors $x\hat{i}-3\hat{j}+7\hat{k}$ and $\hat{i}+y\hat{j}-z\hat{k}$ are collinear, then the value of $\frac{-7xy^2}{z}$ is equal to ____.

19. The range of the function $f(x)=\sqrt{(x-6)}+\sqrt{(12-x)}$ is an interval of length $\sqrt{\lambda}-\sqrt{\mu}$, then $3\lambda - 5\mu$ must be ______.

20. If $S=\sum_{r=1}^{50}\tan^{-1}\left(\frac{2r}{2+r^2+r^4}\right)$, then one's place in the value of 2550 cot S is ______.

◆ ◆

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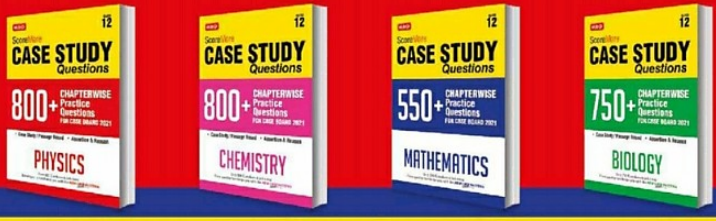
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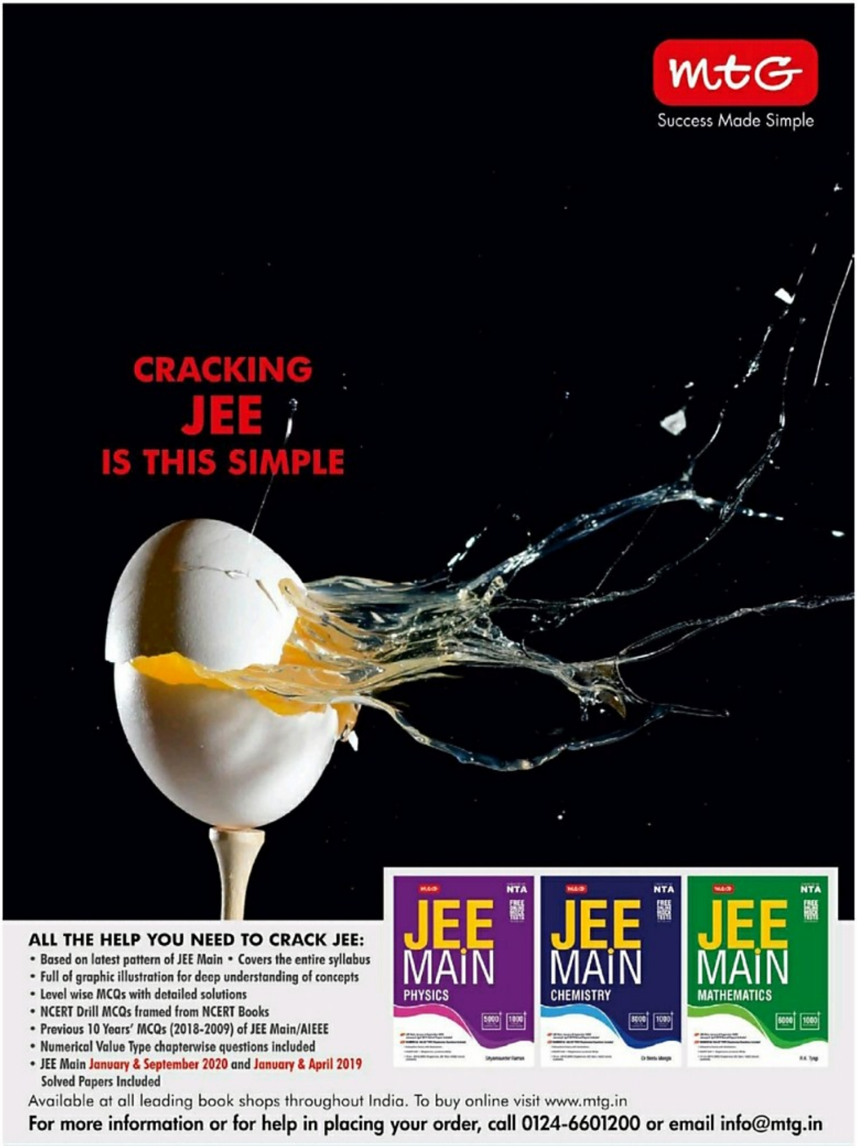
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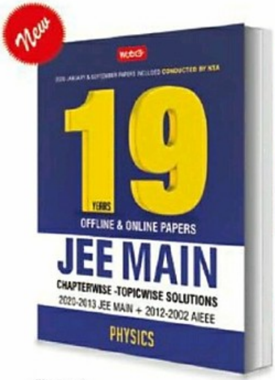
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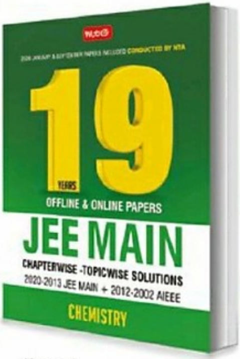
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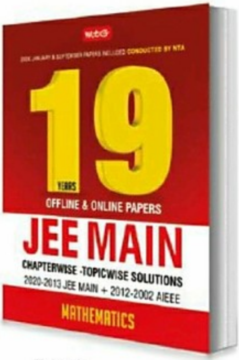
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